

Answers to the Worksheet on Indices and Surds GCSE Non-calculator Miscellaneous

① (a) 25 (b) 25 (c) 1 (d)  $(\sqrt{9})^3 = 3^3 = \underline{\underline{27}}$  (e)  $\frac{1}{2}$   
 (f)  $\frac{1}{5}$  (g)  $\frac{1}{3^2} = \frac{1}{9}$  (h) 2 (i)  $(\sqrt[3]{8})^2 = 2^2 = \underline{\underline{4}}$   
 (j)  $(\sqrt[5]{32})^3 = 2^3 = \underline{\underline{8}}$  (k)  $\frac{1}{(\sqrt[3]{125})^2} = \frac{1}{5^2} = \frac{1}{25}$   
 (l)  $\sqrt[3]{\frac{8}{125}} = \frac{2}{5}$  (m)  $\frac{1}{(\sqrt[3]{\frac{8}{125}})^2} = \frac{1}{(\frac{2}{5})^2} = \frac{1}{\frac{4}{25}} = \frac{25}{4} (=6\frac{1}{4})$   
 (n)  $\frac{1}{\frac{4}{5}} = \frac{5}{4} (=1\frac{1}{4})$  (o)  $\sqrt[3]{\frac{27}{8}} = \frac{3}{2} (=1\frac{1}{2})$

② (a) 2√2 (b) 5√2 (c) 3√2 (d)  $\sqrt{16 \times 3} = \underline{\underline{4\sqrt{3}}}$   
 (e)  $\sqrt{72} = \sqrt{36 \times 2} = \underline{\underline{6\sqrt{2}}}$  (f) 5√5

③ (a) 2<sup>8</sup> (b) a<sup>2</sup> (c) 2<sup>10</sup> (d) 2<sup>10</sup> (e)  $\frac{x^{10}}{x^4} = \underline{\underline{x^6}}$   
 (f)  $\frac{a^6}{a^9} = \underline{\underline{a^{-3}}} (= \frac{1}{a^3})$  (g) x<sup>2</sup> (h) x<sup>-3</sup> (i)  $2^4 \times (\sqrt{3})^4 = 16 \times 9 = \underline{\underline{144}}$   
 (j) x<sup>3</sup> (k)  $(\sqrt[3]{125})^2 = 5^2 = \underline{\underline{25}}$  or  $\sqrt[3]{125 \times 125} = \sqrt[3]{5^3 \times 5^3} = \sqrt[3]{5^6} = 5^2 = \underline{\underline{25}}$   
 (l) 2x<sup>2</sup>y (m)  $(3x^3y^2)^2 = \underline{\underline{9x^6y^4}}$

④ Raise each one to the power of 6  
 $(8^{\frac{1}{2}})^6 = 8^3 = \underline{\underline{512}}$ ,  $(4^{\frac{2}{3}})^6 = 4^4 = \underline{\underline{256}}$ ,  $((2)^{\frac{5}{6}})^6 = 2^5 = \underline{\underline{32}}$   
 $(7^{\frac{1}{3}})^6 = 7^2 = \underline{\underline{49}}$   $(36^{\frac{1}{6}})^6 = \underline{\underline{36}}$   
 Hence:  $2^{\frac{5}{6}}$ ,  $36^{\frac{1}{6}}$ ,  $7^{\frac{1}{3}}$ ,  $4^{\frac{2}{3}}$ ,  $8^{\frac{1}{2}}$

$$5) (a) \frac{\sqrt{2}}{2} \quad (b) \frac{\sqrt{3}}{2} \quad (c) \frac{10}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{10\sqrt{5}}{5} = \underline{\underline{2\sqrt{5}}}$$

$$(d) \frac{\sqrt{132}}{12} \quad (e) \frac{\sqrt{132}}{\sqrt{12}} = \frac{\sqrt{11} \times \sqrt{12}}{\sqrt{12}} = \underline{\underline{\sqrt{11}}}$$

$$(f) \frac{\sqrt{18}}{\sqrt{72}} = \sqrt{\frac{18}{72}} = \sqrt{\frac{1}{4}} = \underline{\underline{\frac{1}{2}}} \quad \text{OR} \quad \frac{\sqrt{18}}{\sqrt{72}} = \frac{\sqrt{2} \times \sqrt{9}}{\sqrt{8} \times \sqrt{9}} = \frac{\sqrt{2}}{2\sqrt{2}} = \underline{\underline{\frac{1}{2}}}$$

$$(g) \frac{\sqrt{80}}{\sqrt{125}} = \frac{\sqrt{16 \times 5}}{\sqrt{25 \times 5}} = \frac{4\sqrt{5}}{5\sqrt{5}} = \underline{\underline{\frac{4}{5}}} \quad \text{OR} \quad \sqrt{\frac{80}{125}} = \sqrt{\frac{16}{25}} = \underline{\underline{\frac{4}{5}}}$$

$$6) (a) 5\sqrt{2} + 2\sqrt{2} - 18\sqrt{2} = \underline{\underline{-11\sqrt{2}}}$$

$$(b) 20 - 12\sqrt{10} + 18 = \underline{\underline{38 - 12\sqrt{10}}}$$

$$(c) 2 + 2\sqrt{10} + 5 = \underline{\underline{7 + 2\sqrt{10}}}$$

$$(d) 6 + 10\sqrt{6} - \sqrt{6} - 10 = \underline{\underline{-4 + 9\sqrt{6}}}$$

$$(e) 12 - 80 = \underline{\underline{-68}} \quad (\text{difference of two squares})$$

$$7) (a) \underline{\underline{8}} \quad (b) \underline{\underline{2\sqrt{5}}} \quad (c) 16 - 5 = \underline{\underline{11}} \quad (d) 16 + 8\sqrt{5} + 5 = \underline{\underline{21 + 8\sqrt{5}}}$$

$$(e) \frac{1}{x} + \frac{1}{w} = \frac{w+x}{xw} = \underline{\underline{\frac{8}{11}}} \quad (\text{using (a) and (c)})$$

$$(f) \frac{1}{(4+\sqrt{5})} \times \frac{(4-\sqrt{5})}{(4-\sqrt{5})} = \frac{4-\sqrt{5}}{16-5} = \underline{\underline{\frac{4-\sqrt{5}}{11}}}$$

$$(g) x^2w^2 = (xw)^2 = 11^2 = \underline{\underline{121}} \quad (\text{using (c)})$$

$$(h) (w-x)^2 = (-2\sqrt{5})^2 = \underline{\underline{20}}$$

$$8) (a) \sqrt{98} = \sqrt{49 \times 2} = \underline{\underline{7\sqrt{2}}}$$

$$(b) P = \sqrt{2} + 7\sqrt{2} = 8\sqrt{2}, \quad P^2 = (8\sqrt{2})^2 = 64 \times 2 = \underline{\underline{128}}$$

$$(c) \frac{1}{\sqrt{98}} = \frac{1}{7\sqrt{2}} = \frac{1}{7} \times \frac{1}{2^{\frac{1}{2}}} = 7^{-1} \times 2^{-\frac{1}{2}}$$

$\underline{\underline{m = -1}}, \quad \underline{\underline{n = -\frac{1}{2}}}$

9) (a)  $2^x = 2^6$   
 $x = 6$

(b)  $4^x = (2^2)^x = 2^{2x} = 32 = 2^5$   
 $2x = 5$   
 $x = \frac{5}{2}$  or  $2.5$

(c)  $9^{2x} = 27$   
 $(3^2)^{2x} = 3^3$   
 $3^{4x} = 3^3$   
 $4x = 3$   
 $x = \frac{3}{4}$

(d)  $\frac{8^x}{2^x} = \frac{(2^3)^x}{2^x} = \frac{2^{3x}}{2^x} = 2^{2x} = 16 = 2^4$   
 $2x = 4$   
 $x = 2$

(e)  $4^{2x} = 2^{4x}$   
 $8^{2x} = 2^{6x}$   
 hence  $\frac{2^{4x}}{2^{6x}} = 64$   
 $2^{-2x} = 2^6$   
 $-2x = 6$   
 $x = -3$

OR  $(\frac{4}{8})^{2x} = 2^6$   
 $(\frac{1}{2})^{2x} = 2^6$   
 $(2^{-1})^{2x} = 2^6$   
 $2^{-2x} = 2^6$   
 $-2x = 6$   
 $x = -3$

10) (a)  $x^2 = (10^m)^2 = 10^{2m}$  (b)  $x^{-1} = (10^m)^{-1} = 10^{-m}$   
 (c)  $xy = 10^m \times 10^{-m} = 10^0 = 1$  (d)  $\frac{x}{y} = \frac{10^m}{10^{-m}} = 10^{2m}$   
 (e)  $\frac{x^3}{(1000)^m} = \frac{10^{3m}}{10^{3m}} = 10^0 = 1$

11) (a)  $10^m \times 10^n = xy$  (b)  $\frac{10^m}{10^n} = \frac{x}{y}$  (c)  $100^m \times 100^n = 10^{2m} \times 10^{2n} = (10^m)^2 \times (10^n)^2 = x^2 y^2$   
 (d)  $10^{2m} \times 10^n = x^2 y$  (e)  $\frac{10^{2m}}{10^n} = \frac{x^2}{y}$  (f)  $10^{-2m} = (10^m)^{-2} = x^{-2}$  (or  $\frac{1}{x^2}$ )

12) (a)  $10^{2m}$  (b)  $10^{-m}$  (c)  $10^{m+n}$  (d)  $10^{m-n}$  (e)  $\frac{10^m}{10^2} = 10^{m-2}$

13)  $m = 125^8 = (5^3)^8 = 5^{24}$   
 $m^{\frac{1}{3}} = (5^{24})^{\frac{1}{3}} = 5^8$   $r = 8$

If you find any errors, please let me know.