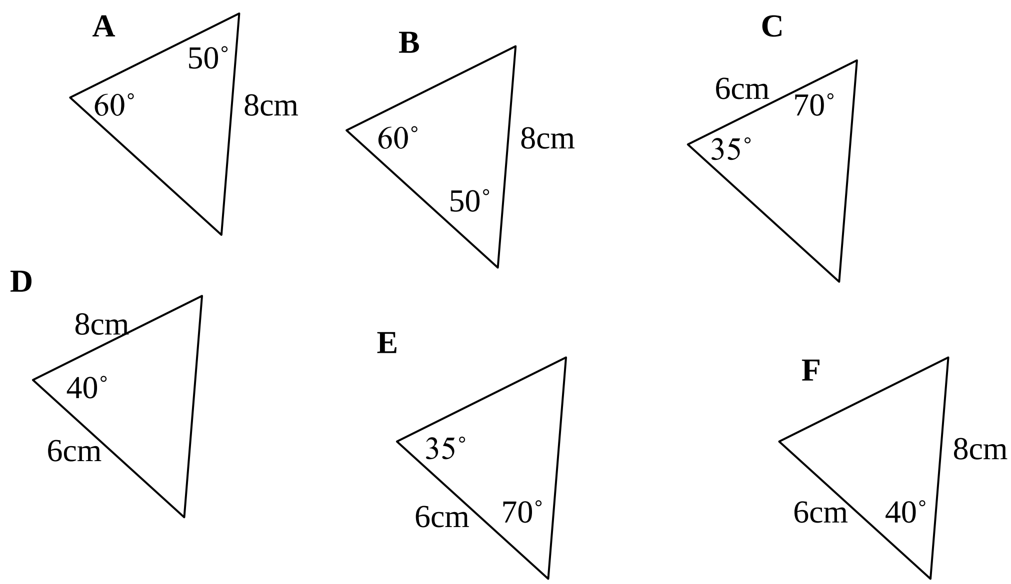
CONGRUENT TRIANGLES NON-CALCULATOR

NOTE: ALL DIAGRAMS **NOT** DRAWN TO SCALE.

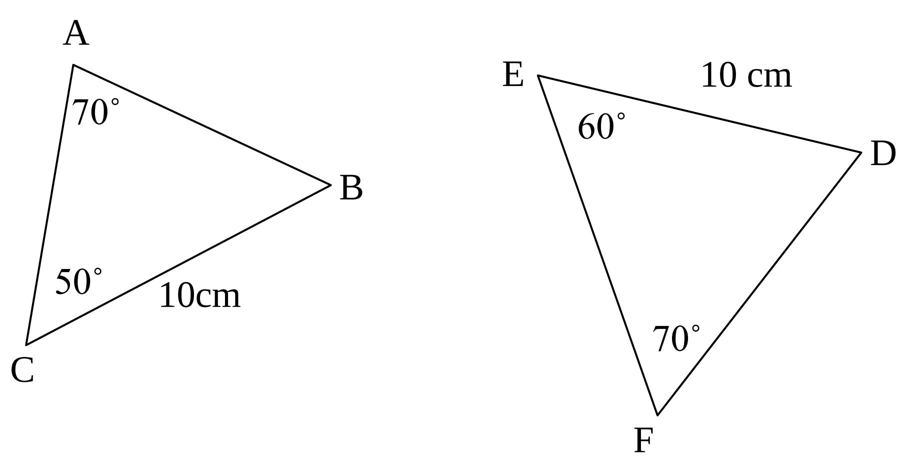
* means "may be challenging for some"

1. Which triangles are congruent? Give reasons.



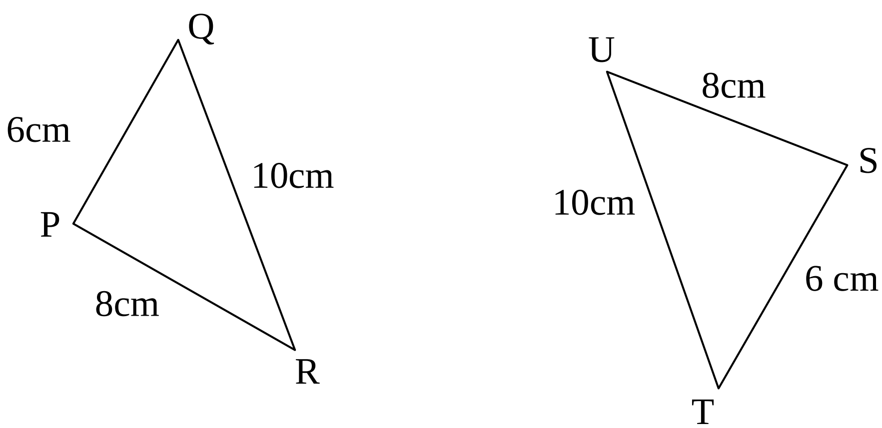
2. Prove that the triangles ABC and DEF are congruent.

Diagrams not drawn to scale



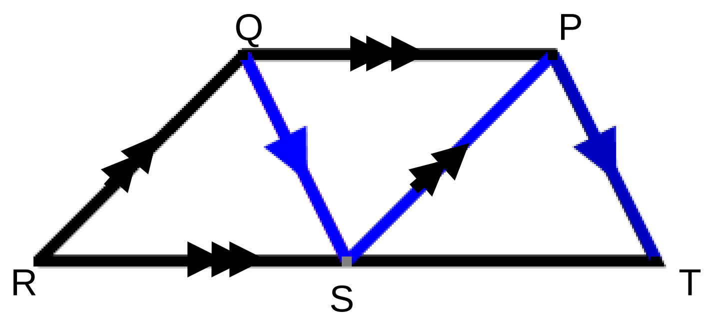
3. Prove that the triangles PQR and STU are congruent.

Diagrams not drawn to scale

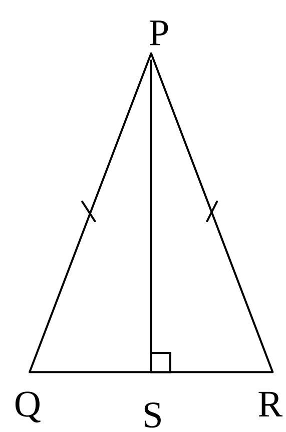


4. PQRS is a parallelogram. The line drawn from P parallel to QS meets RS produced at T.
   Prove that TS = SR.

Diagram not drawn to scale



5. The triangle PQR is an isosceles triangle. PS is perpendicular to QR.
   (a) Use congruent triangles to prove that SQ = SR.
   (b) If PQ = 10cm and QR = 12cm, work out the area of the triangle PQR.



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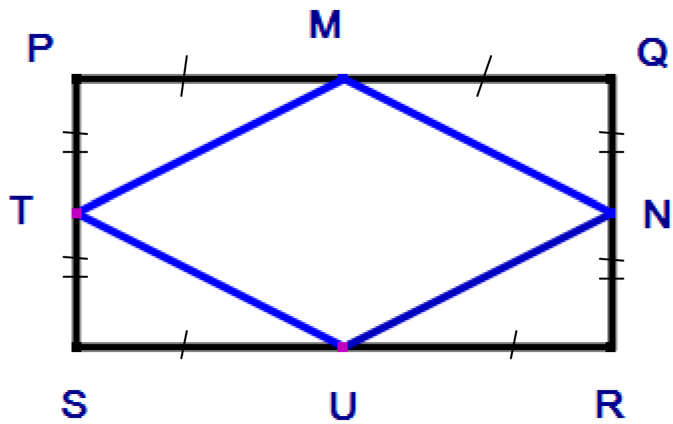
6. PQRS is a rectangle.
M is the mid-point of PQ, N is the midpoint of QR,
T is the midpoint of PS and U is the midpoint of RS.



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(a) Prove that the triangles PMT and RUN are congruent.
(b) Are the lines TM and UN equal? Why?
(c) Is the triangle STU congruent to the triangle RNU? Give reasons
(d) Are the lines TU and UN equal? Why?
(e) What is the special name given to the quadrilateral MNUT?
(f) If PQ = 8cm and PT = 6cm, what is the area of the quadrilateral MNUT?

7. ABCD is a kite, with AB = AD and BC = CD.
(a) Prove that triangles ABC and ADC are congruent.
The line joining B to D meets the diagonal AC at E.
(b) Prove that triangles ABE and ADE are congruent.
(c) Make a geometrical statement about the point E.
(d) If BD = 6cm and AC = 12cm, work out the area of the kite ABCD.

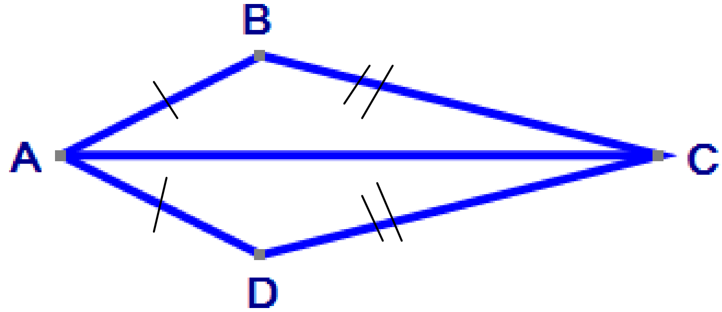
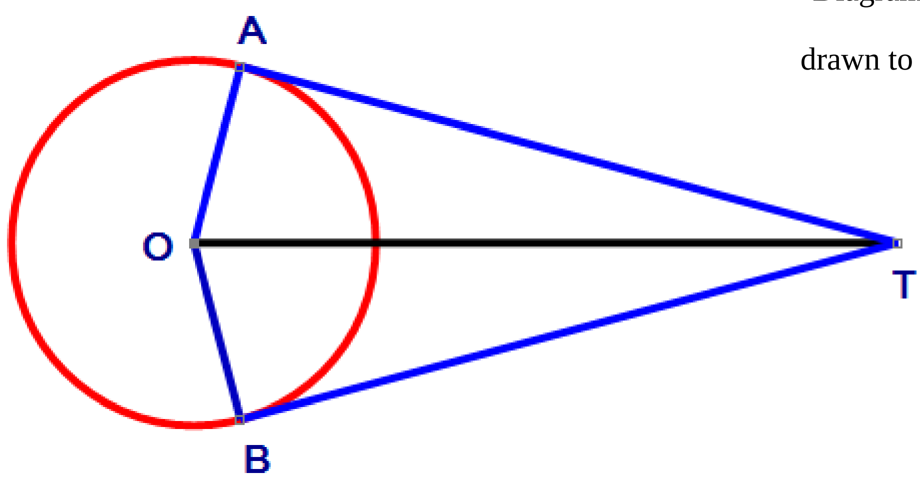


Diagram not drawn to scale

*8. TA and TB are tangents to the circle, centre, O.

(a) Use congruent triangles to prove that AT = BT.
(b) Which angle is equal to angle AOT?
(c) If angle AOT = 40˚, work out the size of angle OTB.
(d) If the radius of the circle is 6cm and OT = 10cm, work out the area of the quadrilateral BOAT.

Diagram not drawn to scale

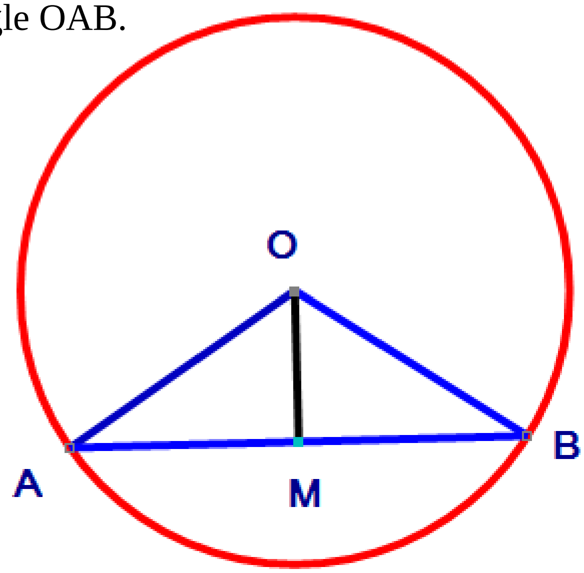


*9. The diagram below shows a circle, centre, O.

AB is a chord of the circle. M is the midpoint of AB.

(a) Use congruent triangles to prove that angle OMA is 90˚.

*(b) If the radius of the circle is 13cm and AB = 24cm, work out the area of triangle OAB.



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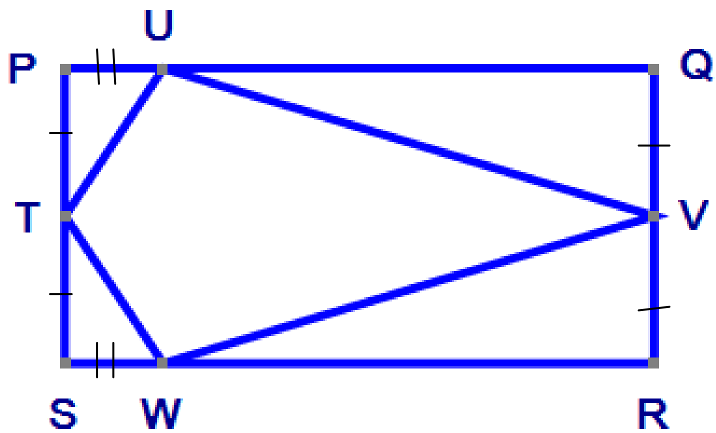
10. PQRS is a rectangle. T and V are the midpoints of PS and QR respectively.

U and W are points on PQ and RS such that PU = SW.

(a) Prove that triangles PUT and SWT are congruent.
(b) Why is UQ = WR?
(c) Hence, or otherwise, prove that triangles QUV and RWV are congruent.
(d) Hence, or otherwise, prove that triangles TUV and TWV are congruent.
(e) What is the special name given to the quadrilateral TUVW?
(f) If PQ = 12cm and QR = 6cm, what is the area of TUVW?
(g) If TV = 20cm and UW = 8cm, what is area of TUVW?



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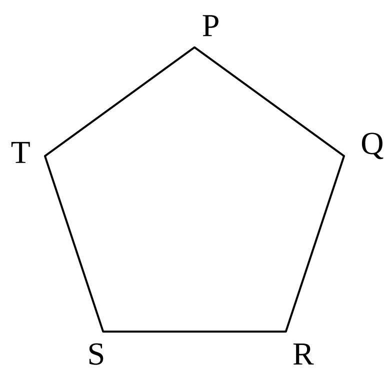
11. PQRST is a regular pentagon.

List all the triangles that are congruent to triangle RTQ.

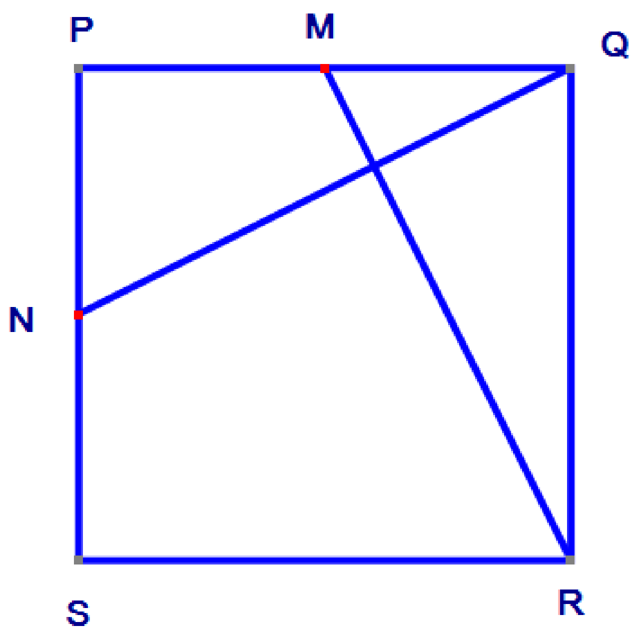


12. PQRS is a square. M is the midpoint of PQ and N is the midpoint of PS.

(a) Use congruent triangles to prove that RM = QN.

* (b) Given that PQ = 2*a*, where *a* is a positive integer,

use Pythagoras' Theorem to show that $\text{RM} = a\sqrt{5}$ = QN.



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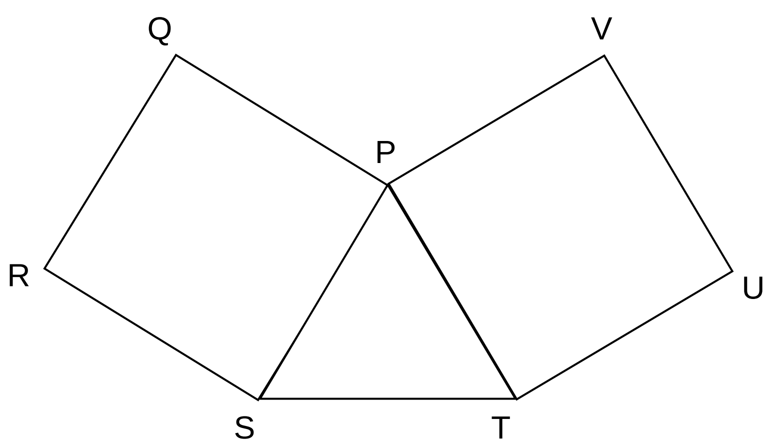
13. PQRS and PTUV are squares attached to the two sides of a triangle.
Prove that: (a) the triangle PQT and PSV are congruent.

** (b) QT is perpendicular to SV (that they meet at 90˚).



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Conditions for Congruent Triangles

1. **SSS** (side, side, side)
   All three corresponding sides are equal in length.
2. **SAS** (side, angle, side)
   A pair of corresponding sides and the included angle are equal.

3. **ASA** (angle, side, angle)
   A pair of corresponding angles and the included side are equal.

4. **AAS** (angle, angle. corresponding side)
   A pair of corresponding angles and a non-included side are equal (the non-included side must be opposite one of the equal angles).

5. **RHS** (Right-angled triangle, hypotenuse, side)
   Two right-angled triangles are congruent if the hypotenuse and one side are equal.

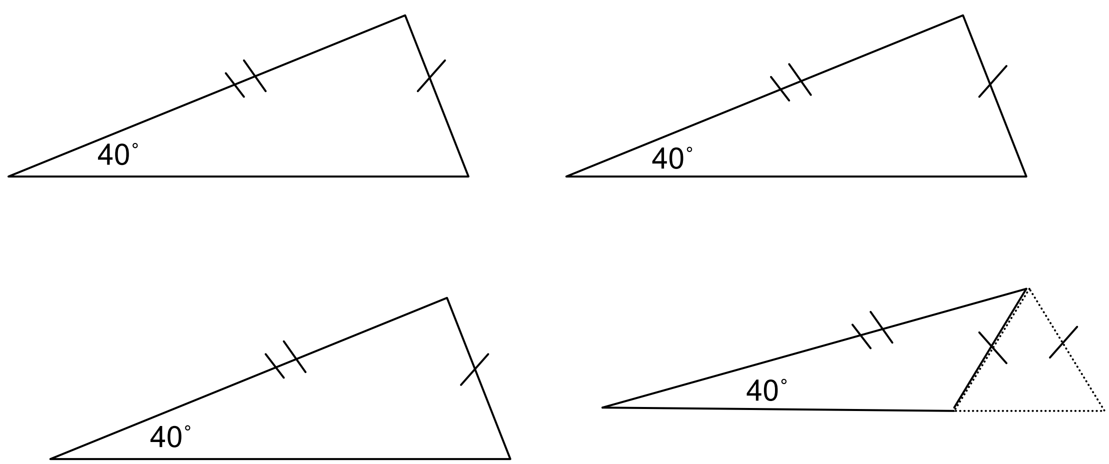
NOTES:

1. AAA does not work.

If all the corresponding angles of a triangle are the same, the triangles will be the same shape, but not necessarily the same size. **The triangles are said to be similar.**

2. SSA also does not work.

Given two sides and a non-included angle, it is possible to draw two different triangles that satisfy the values. It is therefore not sufficient to prove congruence.



Diagrams not drawn to scale

ANSWERS/SOLUTIONS (solutions not unique)

1. A and B by AAS (corresponding side)

   C and D by ASA

   D and F by SAS

2. Angle ABC = 180 – (70 + 50) = 180-120 = 60˚.

   Angle EDF = 180 – (70 + 60) = 180 – 130 = 50˚.

Hence, the triangles are congruent by ASA.

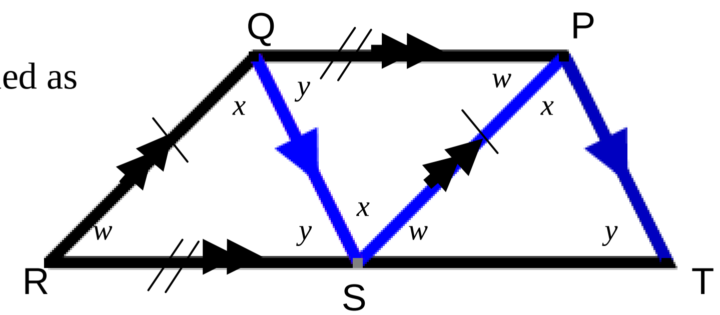
Note: AAS is also acceptable.

3. The three corresponding sides are equal.

   Hence, the triangles are congruent by SSS.

   Furthermore, both triangles are right-angled at P and S respectively, by Pythagoras' Theorem. Hence, RHS and ASA also work.

4. All the angles have been labeled as as shown to help.



Consider the triangles PST and QRS.

Angle SPT = x = QST alternate angles, and angle RQS = x = QSP alternate

Hence, angle SPT = RQS. Also QR = PS (PQRS is a parallelogram) and angle QRS = w = PST (corresponding angles). Hence, by ASA, the triangles PST and QRS are congruent and hence TS = SR. (Note: AAS also works)

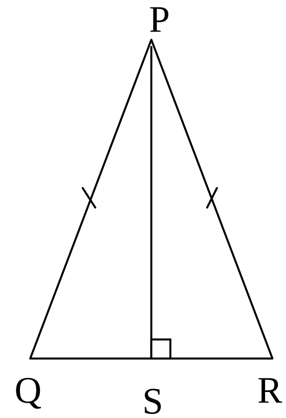
5. Consider triangles PSQ and PSR.

PS = PS common side

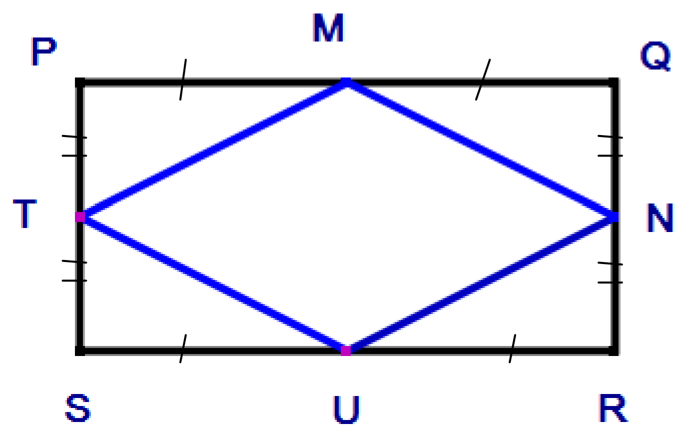
PQ = PR   Isosceles triangle

Angle PSQ = 90˚ = angle PSR

Hence, by RHS the triangles are congruent and hence SQ = SR.



6.



(a) PM = UR, PT = NR   and angle MPT = 90˚ = NRU (rectangle)
    Hence, by SAS the triangles are congruent.

(b) Yes, TM = UN from (a), congruent triangles.

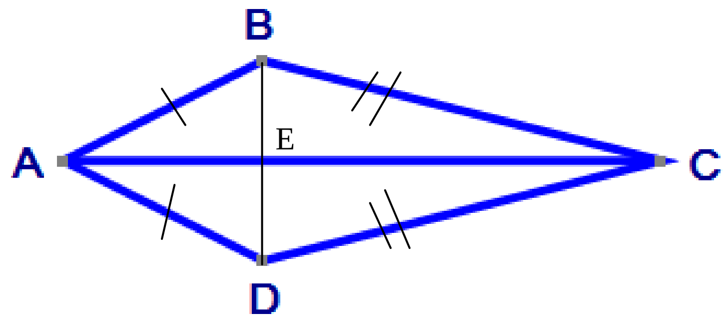
(c) Yes, by SAS

(d) Yes, because of the congruent triangles

(e) Rhombus

(f) Join M to U and T to N. The area of the rhombus is the same as the area of the rectangle.
    Hence, Area = 8 x 6 = 48 cm^2.

7.



(a) AB = AD given
BC = DC given
AC = AC common side
Hence, by SSS the triangles are congruent.

(b) AB = AD given
Angle BAE = DAE from congruent triangles in (a)
AE = AE common side
Hence, the triangles are congruent by SAS

(c) From (b), ED = EB and E is the midpoint of BD.
Note also that angle AED = AEB = 90˚.

(d) If you draw a rectangle around the kite, it becomes easy to see that the area of the kite is half the area of the rectangle.
Hence, area of kite = $\frac{1}{2} \times 12 \times 6 = 36 \text{ cm}^2$.

8. (a) angle OAT = 90˚ = OBT radius and tangent property
OA = OB radii
OT = OT common side
Hence, the triangles OAT and OBT are congruent by RHS,
hence, AT = BT.

(b) Angle BOT.

(c) From the congruent triangles in (a), angle BOT = AOT = 40˚.

Hence, angle OTB = 50˚ angles in the triangle BOT add up to 180˚.

(d) By Pythagoras' theorem, AT = 8cm.
Hence, area of BOAT = 2 × area of triangle AOT

$$= 2 \times \frac{1}{2} \times 6 \times 8 = 48\text{cm}^2$$

9. (a) Consider triangles OMA and OMB.

OA = OB radii

OM = OM common side

MA = MB M is the midpoint of AB.

Hence, by SSS the triangles are congruent and

hence, angle OMA = OMB = 180˚÷ 2 = 90˚ (OMB is a straight line)

(b) AM = 12cm

$OM^2 = 13^2 - 12^2 = 169 - 144 = 25.$

Hence, $OM = 5.$

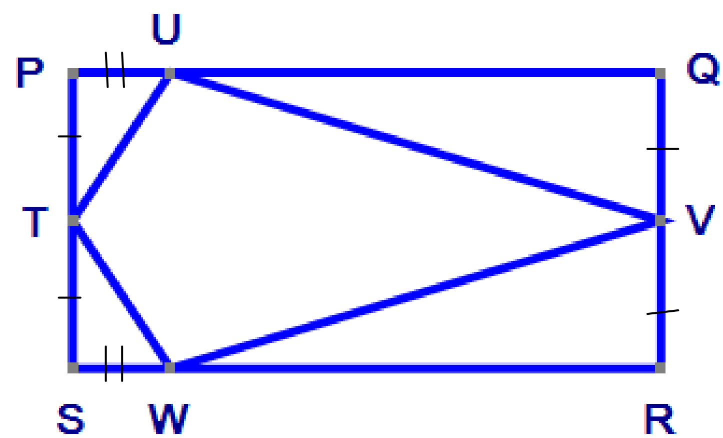
Area of triangle $OAB = \frac{1}{2} \times 24 \times 5 = 60cm^2.$

10. (a) PU = SW given

PT = TS given

Angle UPT = WST = 90˚

Hence, by SAS the triangles are congruent.



(b) PU = SW, PQ = SR

By subtraction, UQ = WR (UQ = PQ – PU)

(c) UQ = WR, QV = VR and angle UQV = WRV = 90˚ rectangle

hence, the triangles are congruent by SAS.

(d) TV = TV common line, TU = TW from (a) and UQ = WR from (b)

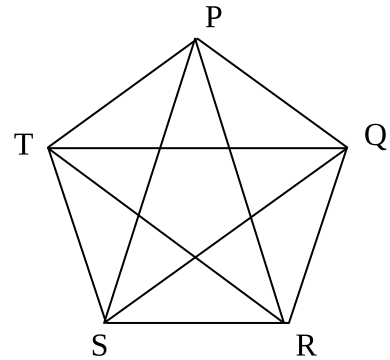
hence, by SSS the triangles are congruent.

(e) TUVW is a kite.

(f) The area of the kite = half the area of the rectangle = $\frac{1}{2} \times 12 \times 6 = 36cm^2.$

(g)The area of the kite = $\frac{1}{2} \times 20 \times 8 = 80cm^2.$

11. QPS, PTR, TSQ and SRP.



12.

(a) Consider triangles

QPN and RQM

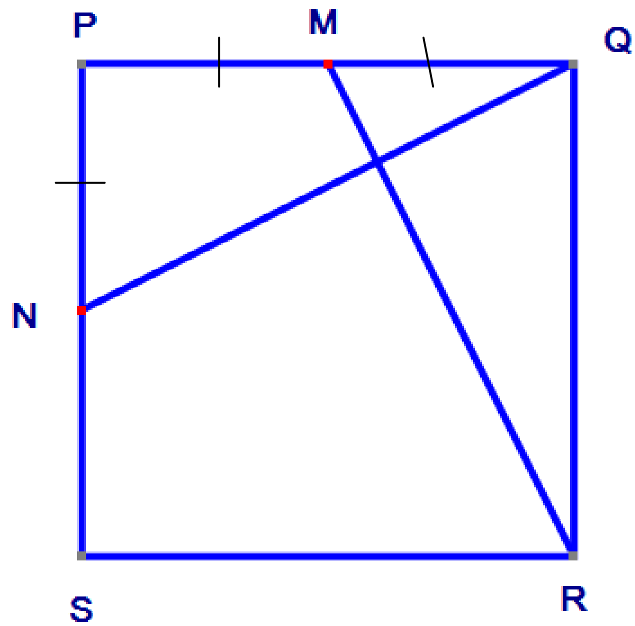
PQ = RQ   square

Angle QPN = 90˚ = RQM

PN = QM   midpoints, M and N

Hence, the triangles are congruent

by SAS and hence RM = QN.

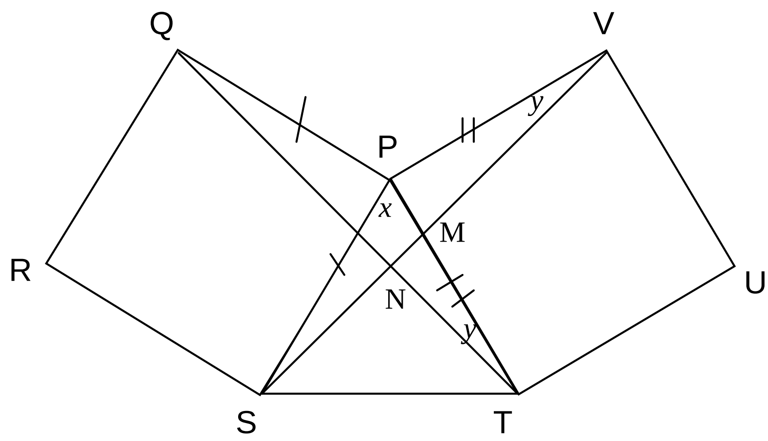


(b) Apply Pythagoras' theorem in triangle QPN

$$QN^2 = (2a)^2 + a^2 = 4a^2 + a^2 = 5a^2$$

$QN = \sqrt{5a^2} = a\sqrt{5}$ = RM   (RM = QN from (a))

13.



(a) Let angle SPT = x.

Triangle PQT	Triangle PSV	Reason
Angle TPQ = x +90	Angle SPV = x + 90	angles in a square = 90
PQ	PS	square
PT	PV	square

Hence, by SAS the triangles are congruent.

(b) Let M be the point of intersection of PT and SV.
Let N be the point of intersection of QT and SV.
Let angle PTQ = y. Then angle SVP = y from the congruent triangles.
In triangle MPV, angle MPV = 90˚ (square)
Hence, angle PMV + y = 90˚. Angles in a triangle add up to 180˚.
Therefore, angle NMT + y = 90˚. Vertically opposite angles.
Hence, angle NMT = 90- y.
In triangle NMT, the three angles add up to 180˚.
So angles MNT + NMT + y = 180
Replace angle NMT by 90 - y,
We get angle MNT + 90 – y + y = 180
Hence, angle MNT = 90˚ and therefore QT is perpendicular to SV.

There is another way of doing (b) and it is on the next page.

(b) Method 2.

Using the exterior angle property:

Angle TMV = 90 + y exterior angle of triangle MPV

But angle TMV is also the exterior angle of triangle TMN.
Hence, angle TMV = 90 + y, which implies that angle MNT = 90˚.