Department for Education

# Mathematics

## GCSE subject content and assessment objectives

**June 2013**

# Contents

# Introduction

GCSE subject criteria set out the knowledge, understanding, skills and assessment objectives common to all GCSE specifications in a given subject. They provide the framework within which awarding organisations create the detail of their specifications, so ensuring progression from key stage 3 national curriculum requirements and the possibilities for development into A level.

## Subject aims and learning outcomes

This document sets out the learning outcomes and content coverage required for a GCSE in mathematics. In subjects such as mathematics, where topics are taught in progressively greater depth over the course of key stage 3 and key stage 4, GCSE outcomes may reflect or build upon subject content which is typically taught at key stage 3. There is no expectation that teaching of such content should be repeated during the GCSE course where it has already been covered at an earlier stage.

GCSE specifications in mathematics should provide a broad, coherent, satisfying and worthwhile course of study. They should encourage students to develop confidence in, and a positive attitude towards, mathematics and to recognise the importance of mathematics in their own lives and to society.

GCSE specifications in mathematics should enable students to:

- develop fluent knowledge, skills and understanding of mathematical methods and concepts
- acquire, select and apply mathematical techniques to solve problems
- reason mathematically, make deductions and inferences and draw conclusions
- comprehend, interpret and communicate mathematical information in a variety of forms appropriate to the information and context.

Students should be aware that mathematics can be used to develop models of real situations and that these models may be more or less effective depending on how the situation has been simplified and the assumptions that have been made. Students should also be able to recall, select and apply mathematical formulae (see Appendix).

# Subject content

GCSE specifications in mathematics should reflect the aims and learning outcomes outlined above, and should include the knowledge, understanding and skills listed below, giving due consideration to the assessment objectives. The essential subject content outlined here provides the framework for developing a coherent study at GCSE.

This content sets out the full range of content for GCSE specifications in mathematics. Awarding organisations may, however, use any flexibility to increase depth, breadth or context within the specified topics or to consolidate teaching of the subject content. **Bold type** identifies content for higher achieving students.

## Scope of study

GCSE specifications in mathematics should require students to:

### Number

1. apply the four operations, including formal written methods, to integers, decimal fractions and simple fractions (proper and improper), and mixed numbers – all both positive and negative
2. apply relationships between operations, including inverse operations, using conventional notation for priority of operations, including brackets, powers, roots and reciprocals
3. calculate with roots, and with integer and **fractional** indices
4. state exactly the result of calculations with fractions, surds and multiples of π; simplify and **rationalise denominators**
5. calculate and interpret with standard form $A \times 10^n$, where $1 \leq A < 10$ and n is an integer

6. apply relationships between fractions and decimal representations, including recurring and terminating decimals
7. apply the relationship between ratio and fractions
8. define percentage as ‘number of parts per hundred’, interpret percentages and percentage changes as fractions or decimals, and calculate these multiplicatively; **apply repeated percentage change**; and solve reverse percentage problems
9. interpret fractions and percentages as operators

10. check calculations using estimation and approximation, including solutions obtained using technology
11. round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures), including simple error intervals using inequality notation

12. apply and interpret limits of accuracy, including **upper and lower bounds**
13. apply the concepts and vocabulary of prime numbers, factor (divisor), multiple, common factors, common multiples, highest common factor and lowest common multiple.

## Algebra

1. interpret and apply algebraic notation
2. manipulate algebraic expressions (including those involving surds and **algebraic fractions**) by:
    - collecting like terms
    - multiplying a single term over a bracket
    - taking out common factors
    - expanding products of two or more binomials
    - factorising quadratic expressions, including the difference of two squares
    - **simplifying expressions involving sums, products and powers, including the laws of indices**
3. argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments **and proofs**

4. derive a formula, in order to solve a problem, then solve the formula

5. **understand and use function notation**
6. **express composition of two familiar functions using function notation**
7. find the inverse of familiar one-to-one functions (e.g. linear functions, reciprocal function, squaring) expressed algebraically

8. apply the conventions for coordinates in the plane and plot points in all four quadrants
9. plot equations that correspond to straight-line graphs in the coordinate plane; apply $y = mx + c$ and the relationship between gradients of parallel and **perpendicular lines**
10. deduce and apply equivalence between algebraic and graphical representations of linear, quadratic, cubic, reciprocal, **exponential and trigonometric relationships**
11. recognise, sketch and produce graphs of linear, quadratic, simple cubic functions, the reciprocal function $y = 1/x$ with $x \neq 0$, **the exponential function $y = k^x$ for positive integer values of k, and the trigonometric functions y = sin x, y = cos x and y = tan x**
12. **sketch translations and reflections of a given function**

13. generate terms of a sequence using term-to-term and position-to-term definitions
14. recognise and use triangle, square and cube numbers, arithmetic progressions and geometric progressions
15. deduce linear **and quadratic** expressions to calculate the nth term of a sequence
16. deduce the sum of an arithmetic series, including where they arise in contextual problems
17. **construct and test conjectures about recursive and long term behaviour of geometric, quadratic and other sequences, including where they arise in contextual problems**

18. construct linear equations in one variable, and solve algebraically and approximately using a graph (including those that require rearrangement)
19. identify and interpret gradients and intercepts of linear functions graphically and algebraically
20. construct quadratic equations and solve algebraically by factorising, **completing the square and using the formula**; and solve approximately by using a graph (**including those that require rearrangement**)
21. identify and interpret gradients, roots, intercepts, turning points of quadratic functions graphically; deduce roots algebraically and **turning points by completing the square**
22. **solve equations numerically using systematic trial and improvement**

23. construct and solve simultaneous equations in two variables (linear/linear or **linear/quadratic**) algebraically, and approximately using a graph
24. solve linear **and quadratic** inequalities in one **or two** variables; represent the solution set on a number line, **in set notation and on a graph**

25. plot and interpret graphs of functions, including piece-wise linear, **exponential and reciprocal graphs**, to approximate solutions to contextual problems such as simple kinematic problems involving distance, velocity and acceleration
26. **calculate or estimate areas under graphs, and interpret results in cases such as velocity-time graphs and graphs in financial contexts**

27. construct, interpret, apply and connect algebraic, graphical and function representations, including in contextual problems.

## Ratio, proportion and rates of change

1. use ratio and scale factor notation, including 1: r where r is a rational number, and apply methods involving conversion, mixing, measuring, scaling, comparing quantities and concentration
2. compare lengths, areas and volumes using ratio notation and scale factors and make links to similarity

3. **construct** and interpret equations that describe direct and inverse proportion
4. recognise and interpret graphs that illustrate direct and inverse proportion

5. apply the concepts of speed, unit pricing and density using compound units
6. **interpret the gradient at a point on a curve as the rate of change, and apply the concepts of instantaneous and average rate of change in graphical representations (chords and tangents)**

7. apply percentage change, including percentage increase, decrease and original value problems, simple interest in financial mathematics, and repeated growth
8. solve growth and decay problems, including **compound interest and use iterative processes**

9. apply the concepts and vocabulary of ratio, direct and inverse proportion and rates of change, represented graphically and algebraically.

## Geometry and measures

1. apply the concepts and vocabulary of mass, length, time, money and other measures

2. derive and apply formulae to calculate:
    - perimeter and area of: triangles, parallelograms, trapezia, circles, and composite shapes, and
    - surface area, cross-sectional area and volume of: cuboids (including cubes), prisms (including cylinders), spheres, pyramids, cones and composite solids
3. measure line segments and angles in geometric figures, including interpreting maps and scale drawings

4. sketch and describe using conventional terms and notations: points, lines, planes, vertices, parallel and perpendicular lines, right angles, and regular, symmetric and irregular plane polygons
5. draw and construct using mathematical tools: parallel and perpendicular lines, right angles, angle bisectors, and regular, symmetric and irregular plane polygons
6. construct and interpret 2D representations of 3D shapes

7. apply the properties and definitions of: special types of quadrilaterals, including square, rectangle, parallelogram, trapezium, kite and rhombus; and triangles and other plane figures using appropriate language
8. apply the properties of: angles at a point, angles at a point on a straight line, perpendicular lines, vertically opposite angles, parallel and intersecting lines, triangles and quadrilaterals, and interior and exterior angles of polygons

9. identify, describe and construct congruent and similar shapes on coordinate axes, by considering rotation, reflection, translation and enlargement (**including negative and fractional scale factors**)
10. identify, describe and construct shapes transformed by stretch parallel to an axis; identify invariant points and lines of each transformation

11. apply the concepts of congruence and similarity, including the relationships between lengths, **areas and volumes** in similar figures
12. apply angle facts, triangle congruence, similarity and properties of named quadrilaterals to derive results and **prove conjectures about angles and sides, using transformational, axiomatic, and property-based deductive reasoning**

13. identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment
14. calculate arc lengths, angles and areas of sectors
15. **apply and prove circle theorems concerning angles, radii, tangents and chords, and apply them within geometric proofs**
16. interpret and express trigonometric relationships algebraically and geometrically
17. apply trigonometric ratios, **sine and cosine rules**, and Pythagoras's theorem in two and **three** dimensions
18. **derive and apply area =½ ab sin C to calculate the area, sides or angles of any triangle**

19. describe translations as 2D vectors

20. apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors; **construct geometric arguments and proofs**
21. **describe the changes and invariance achieved by combinations of rotations, reflections and translations**.

## Probability

1. record and describe the frequency of outcomes of probability experiments using tables and frequency trees
2. apply ideas of randomness, fairness and equally likely events to calculate expected outcomes of multiple future experiments
3. relate relative expected frequencies to theoretical probability, using appropriate language and the 0-1 scale
4. apply the property that the probabilities of an exhaustive set of mutually exclusive outcomes sum to one
5. enumerate sets and combinations of sets systematically, using tables, grids, tree diagrams and Venn diagrams
6. construct theoretical possibility spaces for single and combined events with equally likely and mutually exclusive outcomes and use these to calculate theoretical probabilities
7. calculate the probability of independent and dependent combined events, including tree diagrams and other representations and know the underlying assumptions
8. **calculate and interpret conditional probabilities through representation using two-way tables, tree diagrams, Venn diagrams and by using the formula**
9. understand that empirical samples tend towards theoretical probability distributions, with increasing sample size and with lack of bias
10. interpret risk through assigning values to outcomes (e.g. games, insurance)
11. calculate the expected outcome of a decision and relate to long-run average outcomes.

## Statistics

1. apply statistics to describe a population or a large data set, inferring properties of populations or distributions from a sample, whilst knowing the limitations of sampling
2. construct and interpret appropriate charts and diagrams, including bar charts, pie charts and pictograms for categorical data, and vertical line charts for ungrouped discrete numerical data
3. **construct and interpret diagrams for grouped discrete data and continuous data, i.e. histograms with equal class intervals and cumulative frequency graphs**

4. interpret, analyse and compare univariate empirical distributions through:
    - appropriate graphical representation involving discrete, continuous and grouped data
    - appropriate measures of central tendency, spread and cumulative frequency (median, mean, range, quartiles and inter-quartile range, mode and modal class)
5. describe relationships in bivariate data: sketch trend lines through scatter plots; **calculate lines of best fit**; make predictions; interpolate and extrapolate trends.

# Assessment objectives

| | Assessment objectives | Weighting |
|---|---|---|
| AO1 | ▪ demonstrate accuracy in the use of standard techniques and recall of mathematical knowledge<br>▪ interpret mathematical notation and definitions<br>▪ carry out routine procedures efficiently, with precision, especially in relation to algebra and number. | 35-45%<br><br>At least 40% of the AO1 marks (14-18% of the overall marks) should be within questions which also assess AO2 and/or AO3 |
| AO2 | ▪ reason and communicate accurately, using appropriate terms and correct grammar when developing a mathematical argument (e.g. deduce, justify, generalise, prove)<br>▪ construct substantial chains of reasoning, especially with algebra<br>▪ construct and present clear, logical, convincing arguments, ranging from informal justifications to more rigorous deductive proofs. | 30-40% |
| AO3 | ▪ apply mathematical knowledge and reasoning, linking mathematical ideas and using mathematical modelling to solve problems<br>▪ solve non-routine problems for which an appropriate solution pathway is not immediately evident<br>▪ use and apply concepts and procedures from across content domains<br>▪ make connections between different parts of mathematics<br>▪ solve real world modeling problems that are less well defined, making assumptions and simplifications<br>▪ identify variables and construct relationships between these<br>▪ having formulated problems, solve them, interpreting results and checking them for reasonableness. | 20-30% |

# Appendix 1: Mathematical formulae

(a) Students should be able correctly to recall and apply the following formulae:

(i) Perimeter, area, surface area and volume formulae

$$\text{Perimeter of a circle} = 2\pi r = \pi d$$

$$\text{Area of a circle} = \pi r^2$$

$$\text{Area of a trapezium} = \frac{1}{2}(a+b)h$$

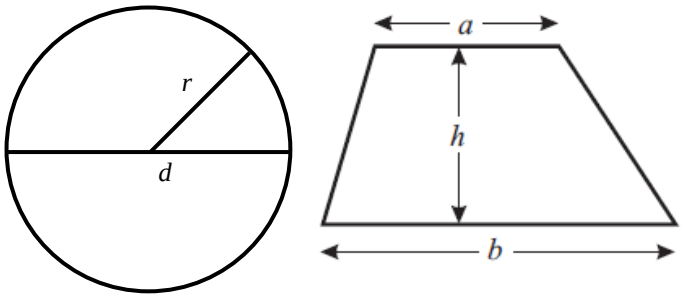
$$\text{Curved surface area of a cone} = \pi r l$$

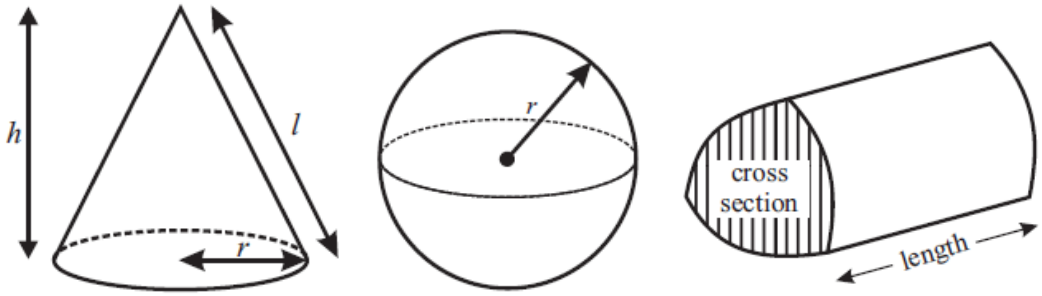
$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Volume of a prism} = \text{area of cross section} \times \text{length}$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$



(ii) Pythagoras's theorem

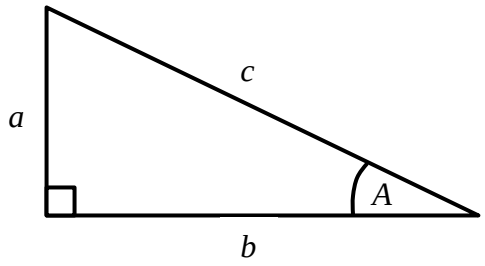
In any right-angled triangle where a, b and c are the length of the sides and c is the hypotenuse:

$$a^2 + b^2 = c^2$$

(iii) Trigonometry formulae

In any right-angled triangle ABC where a, b and c are the length of the sides and c is the hypotenuse:

$$\sin A = \frac{a}{c} \quad \cos A = \frac{b}{c} \quad \tan A = \frac{a}{b}$$

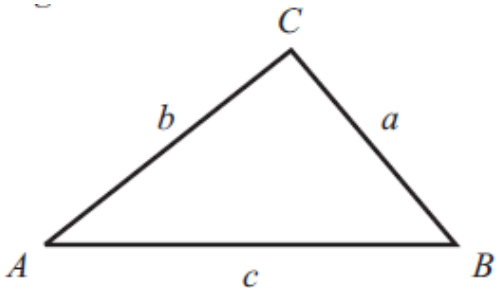



In any triangle ABC:

$$\text{Sine rule } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{Cosine rule } a^2 = b^2 + c^2 - 2bc\cos A$$

$$\text{Area of triangle} = \frac{1}{2}\, a\, b \sin C$$




(iv) Kinematics formulae

$$\text{Velocity} = \frac{\text{distance travelled}}{\text{time taken}}$$

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

(v) The quadratic equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(b) In addition, students should be able correctly to select from a list and apply the following formulae:

(i) Kinematics formulae

Where a is constant acceleration, u is initial velocity, v is final velocity, s is distance travelled and t is time taken:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

(ii) Compound interest

Where P is the principal amount, r is the interest rate over a given period and n is number of times that the interest is compounded:

$$\text{Total accrued} = P\left(1 + \frac{r}{100}\right)^n$$

(iii) Probability

Where P(A) is the probability of outcome A and P(B) is the probability of outcome B:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ and } B) = P(A \text{ given } B)P(B)$$

Department for Education



Any enquiries regarding this publication should be sent to us at: www.education.gov.uk/contactus.

This document is also available from our website at: www.education.gov.uk/schools/teachingandlearning/qualifications/gcses.