QUIZ PAGE 4 Non-Calculator Years 10-13

(Some questions may be accessible to Years 7-9, with hints)

1. If $\frac{98}{19} = e + \frac{1}{f + \frac{1}{g}}$ where *e*, *f* and *g* are all integers, then the value of

   $e + f + g =$ A. 19 B. 14 C. 27 D. 17 E. 15

2. $4^{n+1} + 4^{n+2} =$

   A. 4^{2n+3} B. 8^{2n+3} C. $2 \times 4^{2n+3}$ D. 20×4^{2n} $E.\, 5 \times 2^{2n+2}$

3. Find the value of $(0.4096)^{\frac{3}{4}}$.

4. Simplify $\dfrac{ba^2 - b^3}{ba - b^2} - \dfrac{a^4 - ab^3}{a^3 - ab^2}$

5. Simplify $\sqrt{[(a^2 + b^2)^2 - (a^2 - b^2)^2]}$

6. Find the value of $3^8 - 1$

7. If $a = 1 + 2^n$ and $b = 1 + 2^{-n},$ then $b =$

   A. $\frac{a-2}{a+1}$ **B.** $\frac{a}{a-1}$ C. 3 D. $\frac{a+1}{a-1}$ E. $\frac{a+2}{a+1}$

8. Find the value of $\frac{9^{12}+27^8+81^6}{3^{25}}$

9. What is the unit digit of the answer to:

   $(5^{2006} + 1)(5^{2007} + 1)(5^{2008} + 1)(5^{2009} + 1)(5^{2010} + 1)(5^{2011} + 1)(5^{2012} + 1)$

10. What is the answer to:

$$\frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \frac{1}{4\times 5} + \cdots + \frac{1}{2010\times 2011} + \frac{1}{2011\times 2012}$$

ANSWERS/SOLUTIONS QUIZ PAGE 4

"Solutions not unique"

① $\frac{98}{19} = 5 + \frac{3}{19} = 5 + \frac{1}{\frac{19}{3}} = 5 + \frac{1}{6+\frac{1}{3}}$

$e=5,\ f=6,\ g=3$

$e+f+g = \underline{\underline{14}}$ $\quad\underline{\underline{B}}$

② $4^{n+1} + 4^{n+2} = 4^{n+1}(1+4^1)$

$= 5 \times 4^{n+1} \qquad 4 = 2^2$

$= 5 \times (2^2)^{n+1}$

$= \boxed{5 \times 2^{2n+2}}$

③ $(0.4096)^{\frac{3}{4}} = \left(\frac{4096}{10000}\right)^{\frac{3}{4}} = \left(\frac{4 \times 1024}{10^4}\right)^{\frac{3}{4}}$

$= \left(\frac{2^2 \times 2^{10}}{10^4}\right)^{\frac{3}{4}} = \left(\frac{2^{12}}{10^4}\right)^{\frac{3}{4}} = \frac{2^9}{10^3}$

$= \frac{512}{1000} = \boxed{0.512}$

④ $\frac{ba^2 - b^3}{ba - b^2} - \frac{a^4 - ab^3}{a^3 - ab^2}$

$\boxed{\text{Ans: } \frac{ab}{a+b}}$

$= \frac{\cancel{b}(a^2-b^2)}{\cancel{b}(a-b)} - \frac{\cancel{a}(a^3-b^3)}{\cancel{a}(a^2-b^2)}$

$= \frac{(a+b)\cancel{(a-b)}}{\cancel{(a-b)}} - \frac{\cancel{(a-b)}(a^2+ab+b^2)}{\cancel{(a-b)}(a+b)}$

$= \frac{a+b}{1} - \frac{(a^2+ab+b^2)}{a+b}$

$= \frac{(a+b)^2 - a^2 - ab - b^2}{(a+b)} = \frac{\cancel{a^2}+2ab+\cancel{b^2}-\cancel{a^2}-ab-\cancel{b^2}}{(a+b)}$

$= \underline{\underline{\frac{ab}{a+b}}}$

(5) $$\sqrt{(a^2+b^2)^2-(a^2-b^2)^2}$$

Note p^2-q^2
$=(p+q)(p+q)$
$p=a^2+b^2$
$q=a^2-b^2$

$$=\sqrt{(a^2+b^2+a^2-b^2)(a^2+b^2-(a^2-b^2))}$$

$$=\sqrt{(2a^2)(2b^2)}$$

$$=\sqrt{4a^2b^2}=\boxed{2ab}$$

(6) $$3^8-1$$

$p^2-q^2=(p+q)(p-q)$
$p=3^4 \quad q=1$

$$=(3^4+1)(3^4-1)$$

$$=(82)(3^2+1)(3^2-1) \text{ or } (82)(80)=\underline{\underline{6560}}$$

$$=(82)(10)(8)=656\times 10$$

$$=\underline{\underline{6560}}$$

(7) $a=1+2^n$
$2^n=a-1$

$$b=1+2^{-n}=1+\frac{1}{2^n}=\frac{2^n+1}{2^n}=\underline{\underline{\frac{a}{a-1}}}$$

(8) $$\frac{9^{12}+27^8+81^6}{3^{25}}=\frac{(3^2)^{12}+(3^3)^8+(3^4)^6}{3^{25}}$$

$$=\frac{3^{24}+3^{24}+3^{24}}{3^{25}}=\frac{3\times 3^{24}}{3^{25}}=\frac{3^{25}}{3^{25}}=\underline{\underline{1}}$$

(9) 5^{2006} and all powers of 5 end in a 5. eg $5^3=12\underline{5}$
If you add 1, then $5^{2006}+1$ and all the others will end in a 6. $6\times6\times6\ldots$ will always end in a 6.
hence $\underline{\underline{6}}$ is the unit digit.

(10) express each "fraction" in the form $\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)$ up to $\left(\frac{1}{2011}-\frac{1}{2012}\right)$ hence the sum $=1-\frac{1}{2012}=\underline{\underline{\frac{2011}{2012}}}$

Apologies for the solutions being written by hand. Time is to blame!

I hope you find this useful. If you find any errors, please let me know.