QUIZ PAGE 7 KS3, KS4, Non-Calculator, with Answers/Solutions

1. The integer (whole number) closest to $\frac{14}{3} - \frac{8}{3} \div \frac{4}{5}$ is

   A. 0 B. -1 C. 2 D. 3 E. 1

2. If *n* is an integer, which of the following must be an odd number?

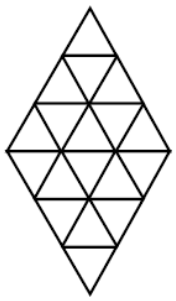
   A. $n-1$ $B.\ n+3$ $C.n^2+7$ $D.\ 2n^2+7$ $E.n^3+1$

3. If *w* is 50% larger than *y* and *x* is 20% larger than *y*, then *w* is what percent larger than *x*?

   A. 25% B. 50% C. 20% D. 125% E. 100%

4. In the diagram, all triangles are equilateral. Work out the total number of equilateral triangles of any size.

   A. 20 B. 28 C. 26 D. 24 E. 18



5. $\frac{3}{7} = 0.\dot{4}28571\dot{}$ is a recurring decimal.

   The 2012th digit after the decimal point is:

   A. 7 B. 5 C. 4 D. 2 E. 1

6. If *n* is an integer (positive or negative whole number including zero), then

   $(n-4),\ (n+6),\ (n+3),\ (n-9)$and $(n-1)$ are also integers.

   If the above five integers are arranged in order from smallest to largest, the median (integer in the middle) is:

   A. $n+9$ B. $n-4$ C. $n-1$ D. $n+3$ E. $n+6$

7. The mean (average) of a set of *n* numbers is 9. When the number -13 is added to the set, the mean is 7. What is the value of *n* ?

   A. 15 B. 13 C. 17. D. 12. E. 10

8. Twenty five tickets are numbered one to twenty five. One ticket is drawn at random (each ticket has an equal chance of being selected). The probability that the ticket drawn is a number that is a multiple of 4 or 5 is:

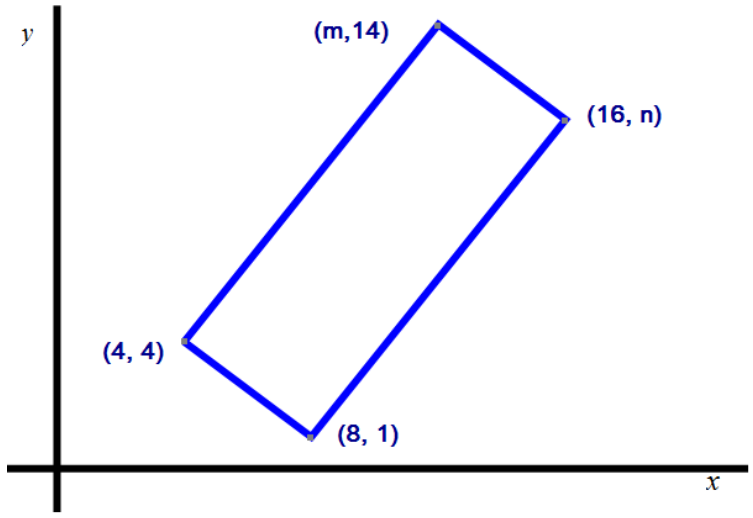
   A. $\frac{8}{25}$ *B*. $\frac{1}{2}$ C. $\frac{11}{25}$ D. $\frac{9}{25}$ E. $\frac{2}{5}$

9. If $2\frac{3}{4}$ of cups of fish food can feed 33 goldfish, how many goldfish would 5 cups feed?

   A. 48 B. 60 C. 12 D. 24 E. 36

10. Given the rectangle below, the value of $m - n$ is:

Diagram not drawn to scale



A. – 1 B. – 4 C. – 2 D. 1 E. 2

11. If I divide my age by 5, the remainder is 2. Your age is three times mine. If I divide your age by 5, what is the remainder?

A. 0 B. 1 C. 2 D. 3 E. 4

12. $\sqrt{\sqrt{\sqrt{144 \times 144 \times 144 \times 144 \times 144 \times 144 \times 144 \times 144}}}$ =

A. 12 B. 20736 C. 144 D. 24 E. 18

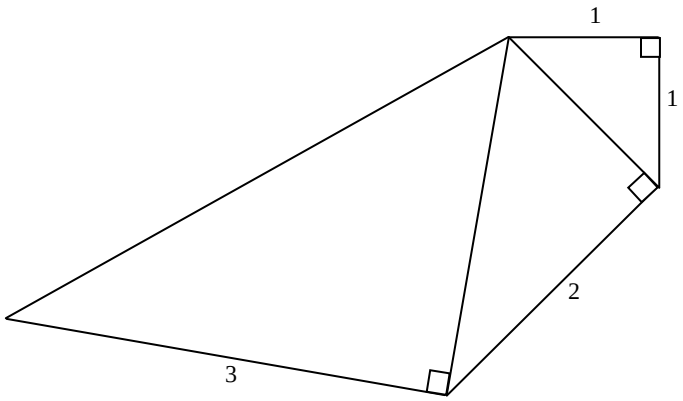
NON- MULTIPLE CHOICE (TEAM WORK)

13. Find all two-digit positive whole numbers such that if you add the two-digit number to the number obtained by reversing its digits, the answer is a perfect square. For example: $29 + 92 = 121 = 11^2$.

    You have been given one of them (29), find the other seven.

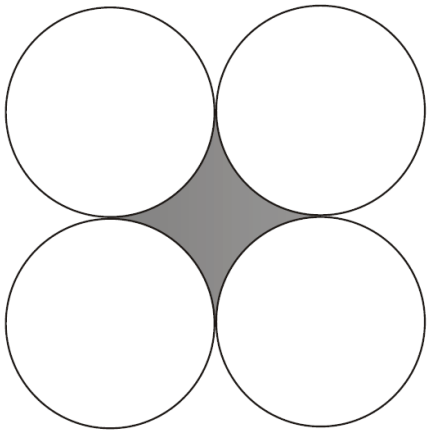
14. If the pattern of 90^o triangles is continued, how many triangles are needed for the hypotenuse to be greater than 14 ?

Diagram not drawn to scale



15. Four coins of radius 1 cm each are arranged as show in the diagram below. Work out the area of the shaded region, leaving your answer in terms of π.

Diagram not drawn to scale



Answers/Solutions (Solutions not unique)

1. The integer (whole number) closest to $\frac{14}{3}-\frac{8}{3}\div\frac{4}{5}$ is

A. 0 B. -1 C. 2 D. 3 E. 1

$$\frac{8}{3}\div\frac{4}{5}=\frac{8}{3}\times\frac{5}{4}=\frac{10}{3},\qquad \frac{14}{3}-\frac{10}{3}=\frac{4}{3}=1\frac{1}{3}$$

Closest integer is 1. **Answers: E**

2. If *n* is an integer, which of the following **must be** an odd number?

A. $n-1$ $B.\ n+3$ $C.n^2+7$ $D.\ 2n^2+7$ $E.n^3+1$

Solution: The only one that must be an odd integer is $2n^2+7$,

since $2n^2$ is always even and even + odd = odd $2n^2+7=$ odd **Answer: D**

3. If *w* is 50% larger than *y* and *x* is 20% larger than *y*, then *w* is what percent larger than *x*?

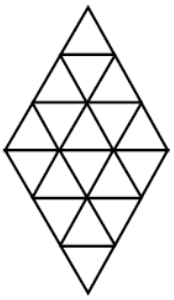
A. 25% B. 50% C. 20% D. 125% E. 100%

Solution: $w = 1.5y$ and $x = 1.2y = 1\frac{2}{10}y=1\frac{1}{5}y=\frac{6}{5}y$

$w=\frac{3}{2}y$ $x=\frac{6}{5}y$ Divide: $\frac{w}{x}=\frac{\frac{3}{2}y}{\frac{6}{5}y}=\frac{3}{2}\times\frac{5}{6}=\frac{5}{4}=1.25=125\%$

Hence *w* is 25% larger than *x*. **Answer: A**

4. In the diagram, all triangles are equilateral. Work out the total number of equilateral triangles of any size.



A. 20 B. 28 C. 26 D. 24 E. 18

Solution:

1by1 equilateral triangles: 18.

2by2 equilateral triangles: 3 at the top half + 3 at the bottom half + 2 overlapping the two

Halves. Total = 8.

3by3 equilateral triangles: 2. Hence total = 18 + 8 + 2 = 28.

Answer: B

5. $\frac{3}{7} = 0.\dot{4}28571\dot{}$ is a recurring decimal.

The 2012^{th} digit after the decimal point is:

A. 7 B. 5 C. 4 D. 2 E. 1

Solution: There are 6 recurring digits. 2012 ÷ 6 = 335 remainder 2.

Hence we would write 335x6 + 2 digits to get the 2012^{th} digit.

Hence the answer is the second digit after the point, 2.

Answer: D

6. If *n* is an integer (positive or negative whole number including zero), then

$(n-4),\ (n+6),\ (n+3),\ (n-9)$and $(n-1)$ are also integers.

If the above five integers are arranged in order from smallest to largest,

the median (integer in the middle) is:

A. $n+9$ B. $n-4$ C. $n-1$ D. $n+3$ E. $n+6$

Solution:

If for example, *n* = 0, the integers are – 4, 6, 3, $-9, -1$.

In order: $-9, -4, -1,\ 3,\ 6$ the median is -1,

which comes from $n-1$. **Answer: C**

OR: the median is the average (mean) in this case.

$$(n-4)+\ (n+6)+(n+3)+(n-9)+(n-1)=5n-5$$

$(5n – 5) \div 5 = n – 1$ **C.**

7. The mean (average) of a set of *n* numbers is 9. When the number -13 is added to the set, the mean is 7. What is the value of *n* ?

A. 15 B. 13 C. 17. D. 12. E. 10

Solution: Sum of *n* numbers = 9*n*, If we add - 13,

then the sum of *n* + 1 numbers is 9*n* – 13.

Hence the average of these *n* +1 numbers = $\frac{9n-13}{n+1}=7,$

hence, $9n-13=7n+7 \Rightarrow 2n=20,\ n=10.$ **Answer: E**

8. Twenty five tickets are numbered one to twenty five. One ticket is drawn at random (each ticket has an equal chance of being selected). The probability that the ticket drawn is a number that is a multiple of 4 or 5 is:

A. $\frac{8}{25}$ *B*. $\frac{1}{2}$ C. $\frac{11}{25}$ D. $\frac{9}{25}$ E. $\frac{2}{5}$

Solution:

The multiples of 4 and 5 are: 4, 5, 8, 10, 12, 15, 16, 20, 24, 25. (10 factors)

Probability $= \frac{10}{25} = \frac{2}{5}$ **Answer: E**

9. If $2\frac{3}{4}$ of cups of fish food can feed 33 goldfish, how many goldfish would 5 cups feed?

A. 48 B. 60 C. 12 D. 24 E. 36

Solution:

$2\frac{3}{4} = \frac{11}{4}$ (11 quarters) cups feed 33,

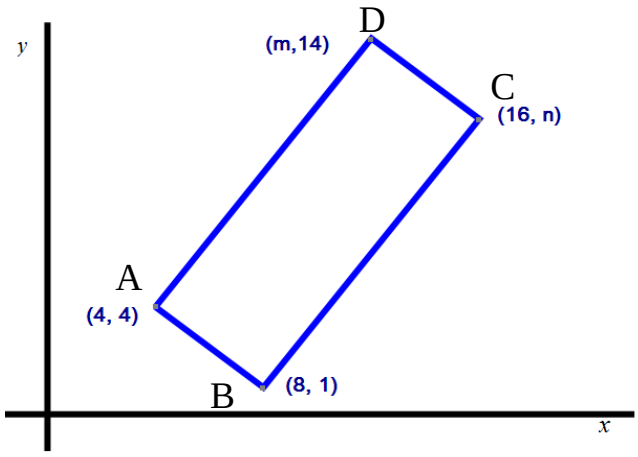
Hence, $\frac{1}{4}$ of a cup feeds $33 \div 11 = 3$ goldfish,

hence $\frac{4}{4} = 1$ cup feeds $4 \times 3 = 12$ goldfish

Hence 5 cups feed $5 \times 12 = 60$ goldfish

Answer: B

10. Given the rectangle below, the value of $m - n$ is:



A. – 1 B. – 4 C. – 2 D. 1 E. 2

Solution: One method is to use translation vectors:

From A (4,4) to B(8,1), the translation vector is $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$

Hence from D(m, 14) to C(16 n), the translation must be the same as A to B.

$\begin{pmatrix} 16 - m \\ n - 14 \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$ Hence m = 12 and n = 11.

Hence $m - n = 1$ **Answer: D**

11. If I divide my age by 5, the remainder is 2. Your age is three times mine.
If I divide your age by 5, what is the remainder?

A. 0 B. 1 C. 2 D. 3 E. 4

Solution: my age may be 7 and then your age would be 3x7 = 21.

21÷ 5 leaves a remainder of 1. **Answer: B**

OR my age may be $5n + 2$ and your age would be $3(5n + 2) = 15n + 6 = 15n + 5 + 1$.

If you divide $15n + 5 + 1$ by 5, the remainder is 1.

12. $\sqrt{\sqrt{\sqrt{144 \times 144 \times 144 \times 144 \times 144 \times 144 \times 144 \times 144}}}$ =

A. 12 B. 20736 C. 144 D. 24 E. 18

Solution: First square root: $\sqrt{144^8} = (144^8)^{\frac{1}{2}} = 144^4$,

Second square root: $\sqrt{144^4} = (144^4)^{\frac{1}{2}} = 144^2$

Third square root: $\sqrt{144^2} = (144^2)^{\frac{1}{2}} = 144$ **Answer: C**

OR $(\mathbf{144^8})^{\frac{1}{8}} = \mathbf{144}$ **C** Note: $\sqrt{\sqrt{\sqrt{n}}} = n^{\frac{1}{8}}$

NON- MULTIPLE CHOICE (TEAM WORK)

13. Find all two-digit positive whole numbers such that if you add the two-digit number to the number obtained by reversing its digits, the answer is a perfect square. For example: 29 + 92 = 121 = 11^2.

You have been given one of them (29), find the other seven.

Solution:

Trial and error will lead to the following answers:

Answers: **38, 47, 56, 65, 74, 83, 92.**

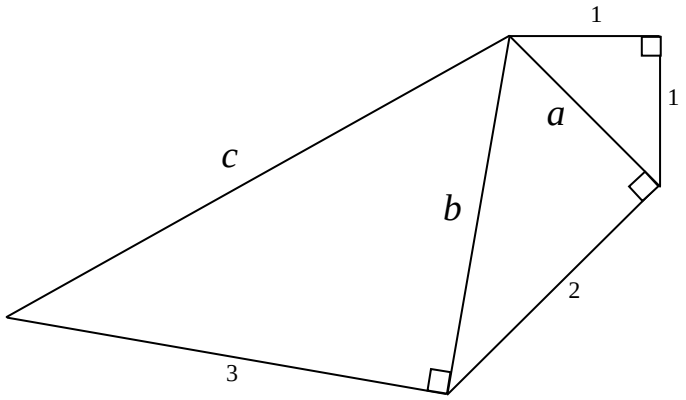
OR: Let the two-digit number be *ab* which is 10*a* + *b* (*a* is the Tens digit)

Reverse the digits, we get *ba* which is worth 10*b* + *a* (*b* is the Tens digit)

Add them together, we get *11a + 11b = 11(a+b).* So we are looking for square numbers that have a factor of 11. Example 121 = 11x11 = 11(2+9), hence 29.

14. If the pattern of 90^o triangles is continued, how many triangles are needed for the hypotenuse to be greater than 14 ?

Diagram not drawn to scale



Solution:

Let the hypotenuses be *a, b, c, d and so on.*

We are looking at the case where say $m^2 > 196$ (*m* being the hypotenuse)

Applying Pythagoras' Theorem to each triangle starting with the top one, we get:

$a^2 = 1^2 + 1^2 = 2$

$b^2 = a^2 + 2^2 = 2 + 4 = 6$

$c^2 = b^2 + 3^2 = 6 + 9 = 15$

$d^2 = c^2 + 4^2 = 15 + 16 = 31$

$e^2 = d + 5^2 = 31 + 25 = 56$

$f^2 = e^2 + 6^2 = 56 + 36 = 92$

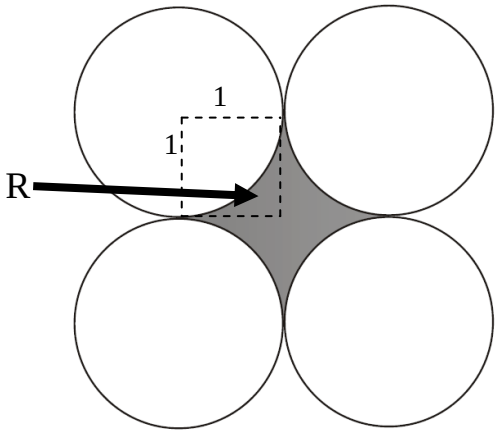
$g^2 = f^2 + 7^2 = 92 + 49 = 141 < 196$

$h^2 = g^2 + 8^2 = 141 + 64 = 205 > 196$

Answer: 8 triangles

15. Four coins of radius 1 cm each are arranged as show in the diagram below.
Work out the area of the shaded region, leaving your answer in terms of π.

Diagram not drawn to scale



Solution:

Consider the one circle and the square shown above.

The area of the shade region labeled R

= area of the square of side 1cm - area of a quarter of the circle

$= 1 \times 1 - \frac{1}{4}\pi \times 1^2$

$= 1 - \frac{\pi}{4}$.

The shaded region between the 4 coins $= 4\left(1 - \frac{\pi}{4}\right)$ *or* $(\mathbf{4} - \boldsymbol{\pi})\boldsymbol{cm^2}$.

OR Join the centres to form a square of side 2cm.

Shaded area = area of the square - area of 4 quarters of a circle (that's one circle)

$= 2 \times 2 - \pi \times 1^2 = (4 - \pi)\text{cm}^2$.

I hope you find this useful. Please let me know if you find any errors.