QUIZ PAGE 5 Non-Calculator KS3 and KS4

1. **Fifty** numbers have an arithmetic mean (average) of 36.

   Two numbers, 67 and 53, are taken away. The mean of the remaining set is

   A. 36.5 B. 32 C. 37.5 D. 52 E. 35

2. The sum of the digits of a six digit number is equal to five.

   What is the value of the product of its digits?

   A. 5 B. 0 C. 6 D. 2 E. 12

3. The pattern 123456123456123456…. is continued to form a 2012 digit number. What is the sum of all 2012 digits?

   A. 2012 B. 7042 C. 7041 D. 7035 E. 7038

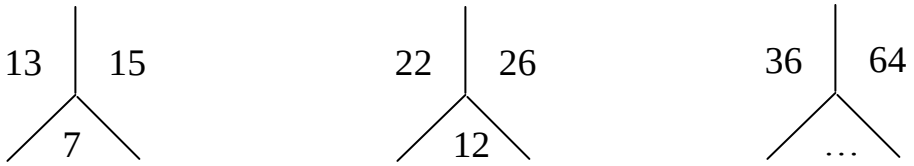
4. The sum of six consecutive positive integers is 513. The smallest number is:

   A. 88 B. 78 C. 86 D. 68 E. 83

5. The sum of 6 consecutive **even** numbers is 330. The largest of the numbers is

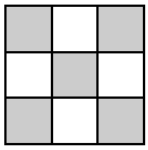
   A. 55 B. 60 C. 48 D. 50 E. 64

6. What is the missing number in the 3rd diagram:

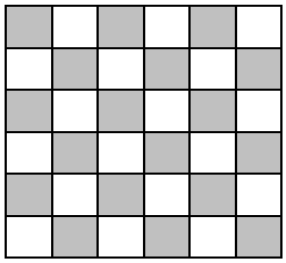


A: 11 B: 14 C: 45 D: 25 E: 17

7. In the diagram, which is a 3 by 3 square, there is a total of 14 squares of all sizes.



What is the total number of squares of all sizes on the 6 by 6 square board below?



A. 63 B. 49 C. 91 C. 105 E. 77

8. a, b, c, d and e are positive real numbers.

If $ab = 2$, $bc = 3$, $cd = 4$, $de = 5$,

What is the value of $\frac{e}{a}$?

A. $\frac{15}{8}$ B. $\frac{5}{6}$ C. $\frac{3}{2}$ D. $\frac{4}{5}$ F. no solution

9. Which one of the following has the largest value?

A. 5×5^5 B. 5^{5^5} C. $(5^5)^5$ D. 5^{55} E. $(5 \times 5)^5$

10. You are given four fractions: $\frac{5}{12}$, p , q and r.

Two fractions p and q are equally spaced between $\frac{5}{12}$ and r .

If $p + q = \frac{4}{3}$, then the value of r is

A. $\frac{7}{12}$ B. $\frac{2}{3}$ C. $\frac{3}{4}$ D. $\frac{5}{6}$ E. $\frac{11}{12}$

11. In the given regular octagon, the size of the angle marked *x*, is:

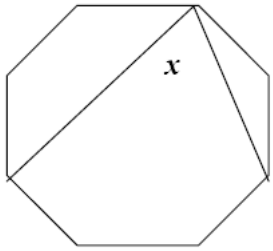


Diagram NOT
Drawn to Scale

A. 22.5˚ B. 45˚ C. 67.5˚ D. 90˚ E. 112.5˚

12. Triangle ABC has D on BC such that BD = 2 and DC = 3.

If AB = m and AD = n,

then the value of $m^2 - n^2$ is

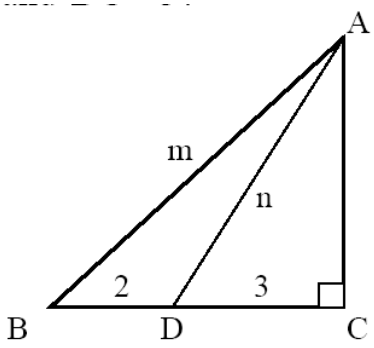


Diagram NOT
Drawn to Scale

A. 4 B. 9 C. 16 D. 25 E. 36

13. When the number 111 222 333 444 555 666 777 888 is divided by 111, **the number of digits** in the quotient (answer) is

A. 9 B. 8 C. 17 D. 25 E. 10

14. When simplified, the product

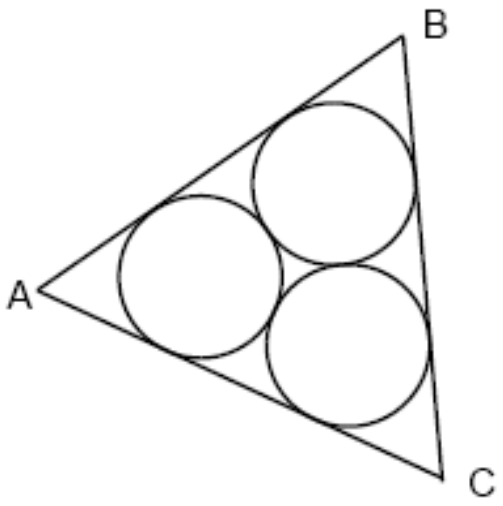
$$(1-\frac{1}{3})(1-\frac{1}{4})(1-\frac{1}{5})...(1-\frac{1}{n})$$ is equal to

A. $\frac{1}{n}$ B. $\frac{2}{n}$ C. $\frac{2(n-1)}{n}$ D. $\frac{2}{n(n+1)}$ E. $\frac{3}{n(n+1)}$

15. Three touching circles, each with radius 1, are inscribed in a triangle ABC.

The exact length of side AB of the triangle is

Diagram NOT
Drawn to Scale



A. 6 B. $3\sqrt{3}$ C. $\frac{\sqrt{3}}{2}$ D. $1+\sqrt{3}$ E. $2+2\sqrt{3}$

Answers/Solutions (Solutions are not unique)

(1) Sum of 50 numbers $= 50 \times 36 = 1800$

$1800 - (67 + 53) = 1800 - 120 = 1680$

Total of 48 numbers $= 1680$

Mean of 48 numbers $= 1680 \div 48$

$= \boxed{35}$

Answer: E

OR

$36 \times 5 = 180$

$\frac{36 \times 10}{2} = 180$

$48\overline{)1680}$ $= 12\overline{)420}$ $= 35$

-36, 60, -60, 00

(2) Since the sum of a six-digit number is 5, one of the digits has to be zero.

Hence the product of the digits $= 0$

Answer: B

(3) $1 + 2 + 3 + 4 + 5 + 6 = 21$

$2012 \div 6 = 335$ R2

hence there are

335 sets of (123456) followed by two numbers 1+2.

$6\overline{)2012}$ $= 335$

-18, 21, -18, 32, -30, 2

$335 \times 20 = 6700$

$6700 + 335 = 7035$

$335 \times 21 + 1 + 2 = 7035 + 1 + 2 = 7038$

Answer: E

(4) $n + (n+1) + (n+2) + (n+3) + (n+4) + (n+5) = 513$

$6n + 15 = 513$

$6n = 498$

$n = 83$

Answer: E

(5) If n is the smallest of the even numbers

then $n + (n+2) + (n+4) + (n+6) + (n+8) + (n+10) = 330$

$6n + 30 = 330$

$6n = 300$

$n = 50$

hence the largest of the numbers $= (n+10) = 60$

Answer: B

(6) diagram 1: $13+15=28$ diagram 2: $22+26=48$

$7=28\div4$ $12=48\div4$

hence the missing number $=(36+64)\div4=100\div4$

$=25$

Answer: D

(7) 3×3 Square $=14$ squares of all sizes $=1^2+2^2+3^2$

4×4 Square $=1^2+2^2+3^2+4^2=30$

5×5 Square $=1^2+2^2+3^2+4^2+5^2=55$

hence

6×6 square $=1^2+2^2+3^2+4^2+5^2+6^2$

$=55+36=91$

Answer: C

(8) $bc=3$, $ab=2$ $\Rightarrow \frac{bc}{ab}=\frac{3}{2}\Rightarrow\frac{c}{a}=\frac{3}{2}\Rightarrow c=\frac{3a}{2}$ —①

$de=5$, $cd=4$ $\Rightarrow \frac{de}{cd}=\frac{5}{4}\Rightarrow \frac{e}{c}=\frac{5}{4}$

but from ① $c=\frac{3a}{2}\Rightarrow \frac{e}{\frac{3a}{2}}=\frac{5}{4}$

$\frac{e}{a}=\frac{5}{4}\times\frac{3}{2}=\frac{15}{8}$

Answer: A

(9) 5^6, 5^{3125}, 5^{25}, 5^{55}, 5^{10}

Largest is 5^{3125} or 5^{5^5}

Answer: B

(10)

$p+q=\frac{4}{3}$

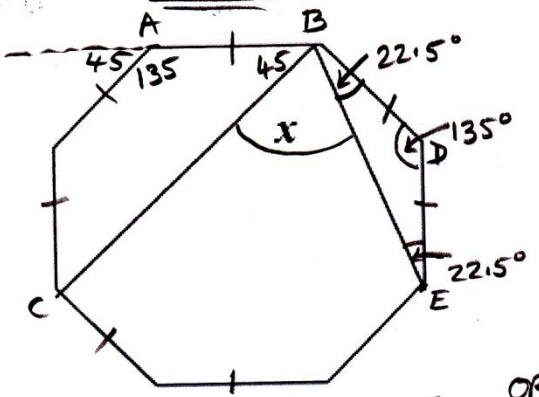
$r-q=p-\frac{5}{12}\Rightarrow r=p+q-\frac{5}{12}$

$r=\frac{4}{3}-\frac{5}{12}=\frac{16}{12}-\frac{5}{12}$

$r=\frac{11}{12}$

Answer: E

11. **Method 1**



Interior angle $= 135°$

$A\hat{B}C = 180 - 135 = 45°$

(Adjacent interior angles between parallel lines are supplementary)

ΔBDE is isosceles

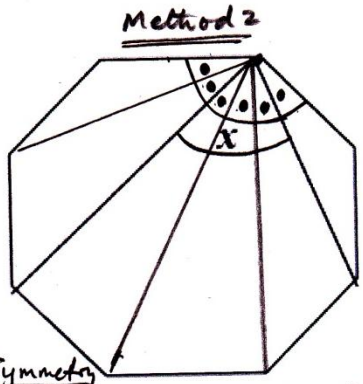
hence $E\hat{B}C = \frac{1}{2}(180 - 135) = \frac{1}{2}(45) = 22.5$

hence $x = 135 - (45 + 22.5)$

$= 135 - 67.5 = 67.5°$

Answer: C

Method 2



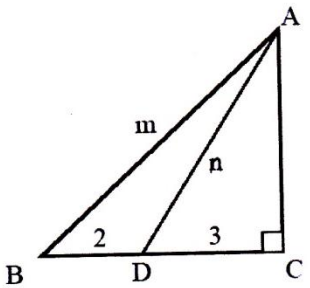
OR use Symmetry

$\blacktriangle = 135 \div 6 = 22.5°$

$x = 3 \times 22.5$

$= 67.5°$

12. Apply Pythagaras' theorem to Δs ABC and ADC respectively.



$\Rightarrow \quad m^2 = 5^2 + AC^2$

and $\quad n^2 = 3^2 + AC^2$

Subtract $\Rightarrow$

$m^2 - n^2 = 5^2 - 3^2$

$m^2 - n^2 = 25 - 9$

$m^2 - n^2 = 16$

Answer: C

(13) $111\overline{)111\,222\,333\,444\,555\,666\,777\,888\,999}$ $\quad 1\,002\,003\,004\,005\,006\,007\,008\,009$

Number of digits in the answer = 25

Answer: D

(14) $\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)\cdots\left(1-\frac{1}{n-1}\right)\left(1-\frac{1}{n}\right)$

$= \frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\times\cdots\times\frac{n-2}{n-1}\times\frac{n-1}{n}$

$= \frac{2}{n}$

Answer: B

(15) Using Trig. OR Pythagoras' Theorem

$\triangle AFD$

$\cos 30 = \frac{AF}{2}$

$AF = 2\cos 30$

$= 2\frac{\sqrt{3}}{2} = \sqrt{3}$

$AF^2 = 2^2 - 1^2 = 3$

$AF = \sqrt{3}$

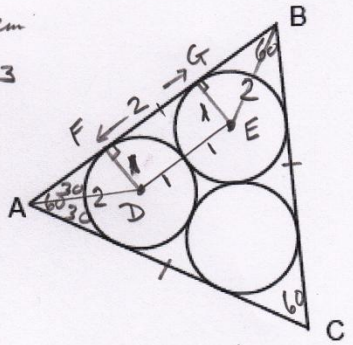
Similarly $BG = \sqrt{3}$

$FG = 2$ (radii 1+1)

hence $AB = AF + 2 + BG$

$= \sqrt{3} + 2 + \sqrt{3}$

$= 2 + 2\sqrt{3}$



Answer: E

I hope you find this useful. If you find any errors, please let me know.