

DIRECTED NUMBERS, TORTURE SQUARES, ADDITION/SUBTRACTION KS3 Non-Calculator

Name: _____



Using number patterns or otherwise, complete the following:

A: ADDITION

+	4	3	2	1	0	-1	-2	-3	-4
4	8	7	6	5	4	3	2	1	0
3		6		4					-1
2		5							
1		4							
0		3							
-1		2							
-2		1							
-3		0							
-4		-1					-6	-7	-8

B: Having completed the torture square, write down the answer to each of the following:

- | | |
|-----------------------|------------------------|
| 1. $4 + (-3) =$ _____ | 5. $-3 + (-4) =$ _____ |
| 2. $-3 + 4 =$ _____ | 6. $-4 + (-3) =$ _____ |
| 3. $2 + (-4) =$ _____ | 7. $-2 + (-3) =$ _____ |
| 4. $-4 + 2 =$ _____ | 8. $-4 + (-1) =$ _____ |
| 9. $-4 + 4 =$ _____ | 10. $0 + (-3) =$ _____ |

11. What do you notice? Write down a rule for adding negative and positive numbers.



12. Apply this rule to get the answer to:

- | | |
|--------------------------------|------------------------------------|
| (a) $-4 + 7 =$ _____ | (m) $-1 + (-1) + 2 =$ _____ |
| (b) $4 + (-7) =$ _____ | (n) $12 + (-15) =$ _____ |
| (c) $-5 + 3 =$ _____ | (o) $-12 + (-8) =$ _____ |
| (d) $-5 + (-3) =$ _____ | (p) $-12 + 9 =$ _____ |
| (e) $-6 + 6 =$ _____ | (q) $9 + (-12) =$ _____ |
| (f) $6 + (-6) =$ _____ | (r) $-9 + (-8) =$ _____ |
| (g) $-8 + (-2) =$ _____ | (s) $-8 + 15 =$ _____ |
| (h) $-8 + 2 =$ _____ | (t) $9 + (-5) + (-4) =$ _____ |
| (i) $-9 + (-9) =$ _____ | (u) $1 + (-2) + 3 + (-4) =$ _____ |
| (j) $-9 + 5 =$ _____ | (v) $8 + (-9) + 10 + (-7) =$ _____ |
| (k) $-9 + (-3) =$ _____ | (w) $-8 + (-9) + 7 + (-6) =$ _____ |
| (l) $-3 + (-2) + (-5) =$ _____ | (x) $-6 + (-8) + 6 + 8 =$ _____ |
| | (y) $0 + (-8) + (-5) + 13 =$ _____ |
| | (z) $-23 + (-27) + 50 =$ _____ |

13. $-1.5 + 2 =$ _____ 14. $-3.5 + (-0.5) =$ _____ 15. $-2.1 + (-3.4) =$ _____

Using number patterns or otherwise, complete the following:



SUBTRACTION

C:

-	6	5	4	3	2	1	0	-1	-2	-3	-4
6	0	1	2	3							
5		0	1	2	3						
4			0	1	2	3					
3				0	1	2	3				
2					0	1	2				
1						0					
0							0				
-1											
-2											
-3											
-4											0

D: Having completed the torture square, write down the answer to each of the following:

1. $4 - (-3) = \underline{\hspace{2cm}}$

5. $-3 - (-4) = \underline{\hspace{2cm}}$

2. $-3 - 4 = \underline{\hspace{2cm}}$

6. $-4 - (-3) = \underline{\hspace{2cm}}$

3. $0 - 4 = \underline{\hspace{2cm}}$

7. $-2 - (-3) = \underline{\hspace{2cm}}$

4. $-4 - (-1) = \underline{\hspace{2cm}}$

8. $-4 - (-1) = \underline{\hspace{2cm}}$

9. $-4 - 4 = \underline{\hspace{2cm}}$

10. $0 - (-3) = \underline{\hspace{2cm}}$

11. What do you notice? Write down a rule for subtracting negative and positive numbers.



12. Apply this rule to get the answer to:

- | | |
|---|--|
| (a) $-4 - 7 = \underline{\hspace{2cm}}$ | (m) $1 - (-1) - 2 = \underline{\hspace{2cm}}$ |
| (b) $4 - 7 = \underline{\hspace{2cm}}$ | (n) $12 - (-15) = \underline{\hspace{2cm}}$ |
| (c) $-5 - 3 = \underline{\hspace{2cm}}$ | (o) $-12 - (-8) = \underline{\hspace{2cm}}$ |
| (d) $-5 - (-5) = \underline{\hspace{2cm}}$ | (p) $-12 - 9 = \underline{\hspace{2cm}}$ |
| (e) $-6 - 6 = \underline{\hspace{2cm}}$ | (q) $9 - 12 = \underline{\hspace{2cm}}$ |
| (f) $-6 - (-6) = \underline{\hspace{2cm}}$ | (r) $-9 - 8 = \underline{\hspace{2cm}}$ |
| (g) $-8 - (-2) = \underline{\hspace{2cm}}$ | (s) $-8 - 15 = \underline{\hspace{2cm}}$ |
| (h) $-8 - 2 = \underline{\hspace{2cm}}$ | (t) $-9 - (-5) - (-4) = \underline{\hspace{2cm}}$ |
| (i) $-9 - (-9) = \underline{\hspace{2cm}}$ | (u) $1 - (-2) - (-3) - 6 = \underline{\hspace{2cm}}$ |
| (j) $-9 - 5 = \underline{\hspace{2cm}}$ | (v) $-8 - (-9) - 10 - (-7) = \underline{\hspace{2cm}}$ |
| (k) $-9 - (-3) = \underline{\hspace{2cm}}$ | (w) $-8 - (-9) - 7 - (-6) = \underline{\hspace{2cm}}$ |
| (l) $-3 - (-2) - (-5) = \underline{\hspace{2cm}}$ | (x) $6 - (-8) - 6 - 8 = \underline{\hspace{2cm}}$ |
| | (y) $0 - 8 - 6 - (-8) = \underline{\hspace{2cm}}$ |
| | (z) $9 - 15 - 8 - (-14) = \underline{\hspace{2cm}}$ |

13. $-1.5 - 2 = \underline{\hspace{2cm}}$ 14. $-3.5 - (-0.5) = \underline{\hspace{2cm}}$ 15. $-2.1 - (-3.4) = \underline{\hspace{2cm}}$

PRACTICE:



Complete the following:

E: ADDITION

+	- 4	-3	- 9	- 10	12	-15	18	-23	14
10	6	7	1	0					
-9		-12							
8		5							
-7									
6									
-5									
4									
-12									
-15							3		-1
-50									

F: SUBTRACTION

-	- 4	-3	- 9	- 10	12	-15	18	-23	14
10	14	13	19	20					
-9		-6	0						
8		11							
-7									
6									
-5									
4									
-12									
-15							-33		-29
-50	-46								

MIXED EXERCISE



G: Write down the answer to each of the following:

1. $-5 - 8 + 13 = \underline{\hspace{2cm}}$

2. $5 - (-8) - 8 = \underline{\hspace{2cm}}$

3. $-5 - 8 + (-8) = \underline{\hspace{2cm}}$

4. $-25 - 5 - (-5) = \underline{\hspace{2cm}}$

5. $-17 + 8 - 5 = \underline{\hspace{2cm}}$

6. $-17 + 8 - (-17) = \underline{\hspace{2cm}}$

7. $-19 - (-8) + (-9) = \underline{\hspace{2cm}}$

8. $-9 - (-9) = \underline{\hspace{2cm}}$

9. $0 - 15 - (-3) + (-3) = \underline{\hspace{2cm}}$

10. $-15 - 3 - 5 - 7 = \underline{\hspace{2cm}}$

11. $5 - 9 - 5 + 9 - 10 + (-10) = \underline{\hspace{2cm}}$

12. $1 + 2 + 3 + 4 + 5 - 1 - 2 - 3 - 4 - 5 = \underline{\hspace{2cm}}$

13. $0 + (-12) - (-15) - (-12) + (-15) = \underline{\hspace{2cm}}$

14. $-99 + (-1) - (-100) - 10 = \underline{\hspace{2cm}}$

Optional: 15. $3x - 5x = \underline{\hspace{2cm}}$ 16. $-2x + 2x = \underline{\hspace{2cm}}$ 17. $-2x - 3x = \underline{\hspace{2cm}}$

18. $-3x - 5x + x = \underline{\hspace{2cm}}$ 19. $-15x - (-7x) = \underline{\hspace{2cm}}$ 20. $-8x + (-3x) = \underline{\hspace{2cm}}$

H: What is the answer to:

$1 - 2 + 3 - 4 + 5 - 6 + 7 - 8 + 9 - 10 + \dots$ up to $+99 - 100 - (-50)$

Can you see a quick way of doing this?

ANSWERS in RED:



A: ADDITION

+	4	3	2	1	0	-1	-2	-3	-4
4	8	7	6	5	4	3	2	1	0
3	7	6	5	4	3	2	1	0	-1
2	6	5	4	3	2	1	0	-1	-2
1	5	4	3	2	1	0	-1	-2	-3
0	4	3	2	1	0	-1	-2	-3	-4
-1	3	2	1	0	-1	-2	-3	-4	-5
-2	2	1	0	-1	-2	-3	-4	-5	-6
-3	1	0	-1	-2	-3	-4	-5	-6	-7
-4	0	-1	-2	-3	-4	-5	-6	-7	-8

B: Having completed the torture square, write down the answer to each of the following:

1. $4 + (-3) = 1$

5. $-3 + (-4) = -7$

2. $-3 + 4 = 1$

6. $-4 + (-3) = -7$

3. $2 + (-4) = -2$

7. $-2 + (-3) = -5$

4. $-4 + 2 = -2$

8. $-4 + (-1) = -5$

9. $-4 + 4 = 0$

10. $0 + (-3) = -3$

11. This is left up to you and your students to decide the best way to do addition.

12.

- | | |
|------------------------------|----------------------------------|
| (a) $-4 + 7 = 3$ | (m) $-1 + (-1) + 2 = 0$ |
| (b) $4 + (-7) = -3$ | (n) $12 + (-15) = -3$ |
| (c) $-5 + 3 = -2$ | (o) $-12 + (-8) = -20$ |
| (d) $-5 + (-3) = -8$ | (p) $-12 + 9 = -3$ |
| (e) $-6 + 6 = 0$ | (q) $9 + (-12) = -3$ |
| (f) $6 + (-6) = 0$ | (r) $-9 + (-8) = -17$ |
| (g) $-8 + (-2) = -10$ | (s) $-8 + 15 = 7$ |
| (h) $-8 + 2 = -6$ | (t) $9 + (-5) + (-4) = 0$ |
| (i) $-9 + (-9) = -18$ | (u) $1 + (-2) + 3 + (-4) = -2$ |
| (j) $-9 + 5 = -4$ | (v) $8 + (-9) + 10 + (-7) = 2$ |
| (k) $-9 + (-3) = -12$ | (w) $-8 + (-9) + 7 + (-6) = -16$ |
| (l) $-3 + (-2) + (-5) = -10$ | (x) $-6 + (-8) + 6 + 8 = 0$ |
| | (y) $0 + (-8) + (-5) + 13 = 0$ |
| | (z) $-23 + (-27) + 50 = 0$ |

13. $-1.5 + 2 = 0.5$

14. $-3.5 + (-0.5) = -4$

15. $-2.1 + (-3.4) = -5.5$

ANSWERS in RED:



SUBTRACTION

C:

-	6	5	4	3	2	1	0	-1	-2	-3	-4
6	0	1	2	3	4	5	6	7	8	9	10
5	-1	0	1	2	3	4	5	6	7	8	9
4	-2	-1	0	1	2	3	4	5	6	7	8
3	-3	-2	-1	0	1	2	3	4	5	6	7
2	-4	-3	-2	-1	0	1	2	3	4	5	6
1	-5	-4	-3	-2	-1	0	1	2	3	4	5
0	-6	-5	-4	-3	-2	-1	0	1	2	3	4
-1	-7	-6	-5	-4	-3	-2	-1	0	1	2	3
-2	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2
-3	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1
-4	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0

D: Having completed the torture square, write down the answer to each of the following:

1. $4 - (-3) = 7$

5. $-3 - (-4) = -1$

2. $-3 - 4 = -7$

6. $-4 - (-3) = -1$

3. $0 - 4 = -4$

7. $-2 - (-3) = 1$

4. $-4 - (-1) = -3$

8. $-4 - (-1) = -3$

9. $-4 - 4 = -8$

10. $0 - (-3) = 3$

ANSWERS in RED:



11. **For example:** $-(-) = +$ as in $-3 - (-2) = -3 + 2$
 $-3 - 5 = -8$: add 3 to 5, both negative, hence, -8
 $-8 + 5$: take the smaller one (ignore signs) from the larger
we get 3. The sign is the sign of the larger one (-).
Hence, $-8 + 5 = -3$.

It is left up to you and your students to decide the best way to do subtraction. Number lines may play a part here.

12. Apply this rule to get the answer to:

- (a) $-4 - 7 = -11$ (m) $1 - (-1) - 2 = 0$
(b) $4 - 7 = -3$ (n) $12 - (-8) = 20$
(c) $-5 - 3 = -8$ (o) $-12 - (-8) = -4$
(d) $-5 - (-5) = 0$ (p) $-12 - 9 = -21$
(e) $-6 - 6 = -12$ (q) $9 - 12 = -3$
(f) $-6 - (-6) = 0$ (r) $-9 - 8 = -17$
(g) $-8 - (-2) = -6$ (s) $-8 - 15 = -23$
(h) $-8 - 2 = -10$ (t) $-9 - (-5) - (-4) = 0$
(i) $-9 - (-9) = 0$ (u) $1 - (-2) - (-3) - 6 = 0$
(j) $-9 - 5 = -14$ (v) $-8 - (-9) - 10 - (-7) = -2$
(k) $-9 - (-3) = -6$ (w) $-8 - (-9) - 7 - (-6) = 0$
(l) $-7 - (-2) - (-5) = 0$ (x) $6 - (-8) - 6 - 8 = 0$
(y) $0 - 8 - 6 - (-8) = -6$ (z) $9 - 15 - 8 - (-14) = 0$

13. $-1.5 - 2 = -3.5$ 14. $-3.5 - (-0.5) = -3$ 15. $-2.1 - (-3.4) = 1.3$

PRACTICE: ANSWERS in RED:



Complete the following:

E: ADDITION

+	- 4	-3	- 9	- 10	12	-15	18	-23	14
10	6	7	1	0	22	-5	28	-13	24
-9	-13	-12	-18	-19	3	-24	9	-32	5
8	4	5	-1	-2	20	-7	26	-15	22
-7	-11	-10	-16	-17	5	-22	11	-30	7
6	2	3	-3	-4	18	-9	24	-17	20
-5	-9	-8	-14	-15	7	-20	13	-28	9
4	0	1	-5	-6	16	-11	22	-19	10
-12	-16	-15	-21	-22	0	-27	6	-35	2
-15	-19	-18	-24	-25	-3	-30	3	-38	-1
-50	-54	-53	-59	-60	-38	-65	-32	-73	-36

F: SUBTRACTION

-	- 4	-3	- 9	- 10	12	-15	18	-23	14
10	14	13	19	20	-2	25	-8	33	-4
-9	-5	-6	0	1	-21	6	-27	14	-23
8	12	11	17	18	-4	23	-10	31	-6
-7	-3	-4	2	3	-19	8	-25	16	-21
6	10	9	15	16	-6	21	-12	29	-8
-5	-1	-2	4	5	-17	20	-23	18	-19
4	8	7	13	14	-8	19	-14	27	-10
-12	-8	-9	-3	-2	-24	3	-30	11	-26
-15	-11	-15	-6	-5	-27	0	-33	8	-29
-50	-46	-47	-41	-40	-62	-35	-68	-27	-64

MIXED EXERCISE: ANSWERS in RED:



G:

1. $-5 - 8 + 13 = 0$

2. $5 - (-8) - 8 = 5$

3. $-5 - 8 + (-8) = -21$

4. $-25 - 5 - (-5) = -25$

5. $-17 + 8 - 5 = -14$

6. $-17 + 8 - (-17) = 8$

7. $-19 - (-8) + (-9) = -20$

8. $-9 - (-9) = 0$

9. $0 - 15 - (-3) + (-3) = -15$

10. $-15 - 3 - 5 - 7 = -30$ $(-15 - 5 = -20)$. $(-3 - 7 = -10)$

11. $5 - 9 - 5 + 9 - 10 + (-10) = -20$

12. $1 + 2 + 3 + 4 + 5 - 1 - 2 - 3 - 4 - 5 = 0$

13. $0 + (-12) - (-15) - (-12) + (-15) = 0$

14. $-99 + (-1) - (-100) - 10 = -10$

Optional: 15. $3x - 5x = -2x$ 16. $-2x + 2x = 0$ 17. $-2x - 3x = -5x$

18. $-3x - 5x + x = -7x$ 19. $-15x - (-7x) = -8x$ 20. $-8x + (-3x) = -11x$

H: What is the answer to:

$$1 - 2 + 3 - 4 + 5 - 6 + 7 - 8 + 9 - 10 + \dots \text{up to } + 99 - 100 - (-50) = 0$$

$$1 - 2 = -1, 3 - 4 = -1, 5 - 6 = -1, 50 \text{ lots of } -1 = -50.$$

$$-50 - (-50) = -50 + 50 = 0$$

I hope you find this useful and there are no errors!